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① $f(n) = 2(7^n) + 3(5^n)$ எனக, $n = 1$ ஆக $f(1) = 29$

$f(1) = 24(1) + 5$ ∴ முடிவு உண்மை

$n = p$ இற்கு முடிவு உண்மை எனக; $p \in \mathbb{Z}^+$

$F(p) = 2(7^p) + 3(5^n) = 24(k) + 5$; $k \in \mathbb{Z}^+$

$$f(p+1) = 2(7^{p+1}) + 3(5^{p+1}) = 14(7^p) + 15(5^p)$$
$$= 2(7^p) + 3(5^p) + 12[7^p + 5^p]$$
$$= 24(k) + 24\left[\tfrac{1}{2}(7^p + 5^p)\right] + 5$$
$$F(p+1) = 24\left(k + \tfrac{1}{2}[7^p + 5^p]\right) + 5 \;;\; \left[k + \tfrac{1}{2}(7^p + 5^p)\right] \in \mathbb{Z}^+$$

∴ $n = p+1$ இற்கு முடிவு உண்மை.

∴ $\forall n \in \mathbb{Z}^+$ இற்கும் கணிதத் தொகுத்தறிவு முறை மூலம் முடிவு உண்மை ஆகும்.

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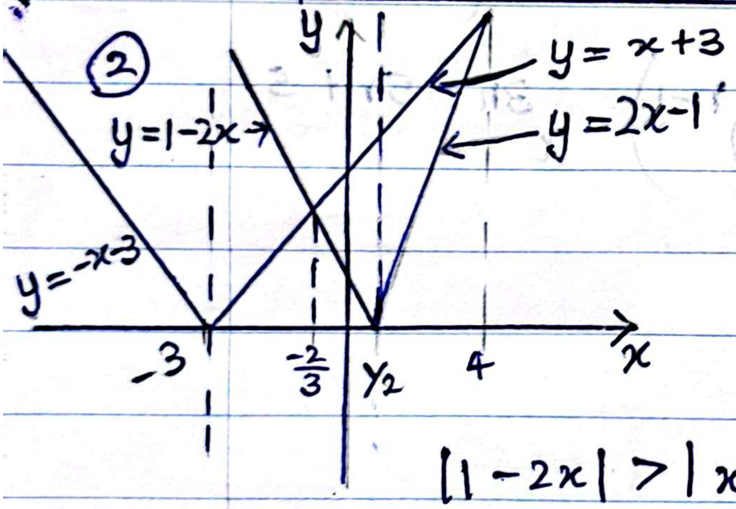
②



$x + 3 = 2x - 1$
$x = 4$

$x + 3 = 1 - 2x$
$x = \left(-\frac{2}{3}\right)$

$|1 - 2x| > |x + 3|$ ஐ திருப்தி செய்யும்

$x \Rightarrow$

$x > 4$ or $x < -\frac{2}{3}$

Isuru

$$\sqrt{3}a - 1 + (a+\sqrt{3})i = 2(a-i)$$

(3) $\sqrt{3} + a = -2 \Rightarrow a = -(2+\sqrt{3})$,

$$z_1 = \frac{2+\sqrt{3}-i}{2+\sqrt{3}+i} = \frac{(2+\sqrt{3}-i)^2}{(2+\sqrt{3})^2+1} = \frac{4+3-1+4\sqrt{3}-2\sqrt{3}i-4i}{4+3+1+4\sqrt{3}}$$

$$z_1 = \frac{2\sqrt{3}(\sqrt{3}+2)}{4(\sqrt{3}+2)} - \frac{2i(\sqrt{3}+2)}{4(\sqrt{3}+2)} = \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)i$$

$$= \cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)$$

$\arg(z_1) = \frac{-\pi}{6}$ $\quad |z_1| = 1$

(4) $$\left(x^3 - \frac{1}{x^2}\right)^n = {}^nC_{r-1} \times (x^3)^{n-r+1} \times (-x^{-2})^{r-1}$$

$$= \left({}^nC_{r-1} \times (-1)^{r-1}\right) x^{3n-5r+5}$$

$3n - 5r + 5 = t$

$(3n - t) = 5r - 5$

$= 5(r-1)$

$\therefore (3n-t)$ ஆனது

5 இன் மடங்கு , $r \geq 1$

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⑤ $\lim_{x\to 0} \frac{\tan(ax^2)}{x^2} = \lim_{x\to 0} \cdot \frac{\sin(ax^2)}{x^2 \cos(ax^2)}$

$= a \lim_{ax^2\to 0} \frac{\sin(ax^2)}{ax^2} \times \lim_{x\to 0} \frac{1}{\cos(ax^2)}$

$= a(1) \times (1)$

$= a$

$\lim_{x\to 0} \frac{\tan 7x^2 + \tan 8x^2}{\sin^2 x}$

$= \frac{\lim_{x\to 0} \frac{\tan 7x^2}{x^2} + \lim_{x\to 0} \frac{\tan 8x^2}{x^2}}{\left(\lim_{x\to 0} \frac{\sin x}{x}\right)^2}$

$= \frac{(7) + (8)}{(1)^2}$

$= 15$

⑥ $V = \int \pi y^2 dx$

$V = \pi \int_0^1 \frac{3x+1-1}{(3x+1)^2} dx$

$= \pi \int_0^1 \frac{1}{3x+1} - \frac{1}{(3x+1)^2} dx$

$= \pi \left[\frac{\ln(3x+1)}{3} - \frac{(3x+1)^{-1}}{(-1)(3)} \right]_0^1$

$= \pi \left[\frac{\ln(3x+1)}{3} + \frac{1}{3(3x+1)} \right]_0^1$

$V = \pi \left[\left(\frac{\ln 4}{3} + \frac{1}{12} \right) - \left(0 + \frac{1}{3} \right) \right]$

$= \pi \left[\frac{8\ln 2 - 3}{12} \right]$

$= \frac{\pi}{12} \left[8\ln 2 - 3 \right]$

Isuru

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(7) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\left(\dfrac{dy}{dx}\right) = \dfrac{-xb^2}{ya^2}$

$\left[\dfrac{dy}{dx}\right]_{\substack{x = a\cos\theta \\ y = b\sin\theta}} = \dfrac{-a\cos\theta\, b^2}{b\sin\theta\, a^2}$

தொடலி ⇒

$-\dfrac{b\cos\theta}{a\sin\theta} = \dfrac{y - b\sin\theta}{x - a\cos\theta}$

தொடலி ⇒

$\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$ — (1)

(2a,0) தொடலியில் உள்ளதால்
(2a,0), (1) ஐ திருப்தி செய்யும்

$2\cos\theta = 1$

$\theta = \pi/3 \quad (0 < \theta < \pi/2)$

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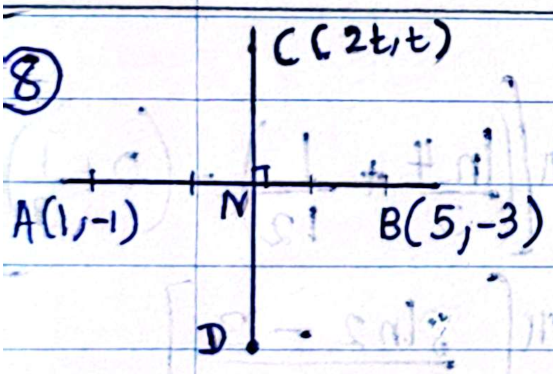
(8)



$N \equiv (3,-2)$ $\quad M_{AB} = \dfrac{-3+1}{5-1}$

$= \dfrac{-2}{4}$

$= \dfrac{-1}{2}$

$M_{AB} \times M_{CN} = (-1)$

$M_{CN} = 2$

CN ⇒

$2 = \dfrac{y-(-2)}{x-3}$

$2x - y - 8 = 0$

$t = \frac{8}{3}$, $C \equiv \left(\frac{16}{3}, \frac{8}{3}\right)$

N, CD இன் நடுப்புள்ளி,

$D \equiv \left(\dfrac{-7}{3}, \dfrac{-14}{3}\right)$

Jsuru

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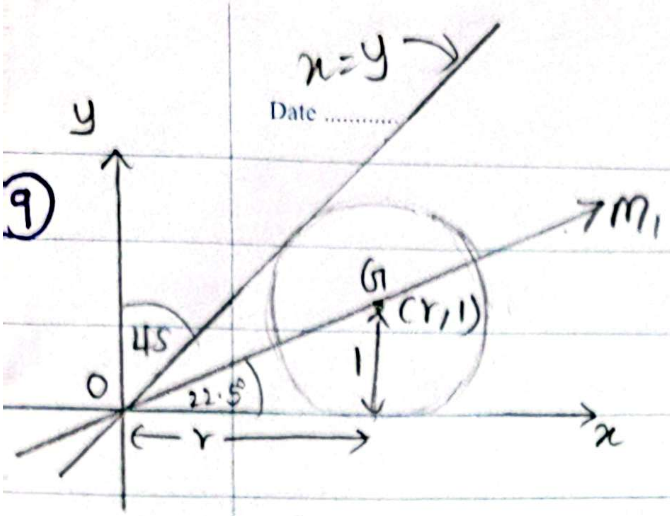

(9)

$$\tan 45 = \frac{2\tan(22.5)}{1-\tan^2(22.5)}$$

$$1 = \frac{2m_1}{1-m_1^2}$$

$$m_1 = \pm\sqrt{2} - 1$$

$$m_1 = \sqrt{2}-1 \quad (m_1 > 0)$$

$OG \Rightarrow y = (\sqrt{2}-1)x$

இதை $(r,1)$ திருப்தி செய்யும்

$\Rightarrow r = (\sqrt{2}+1)$

வேண்டிய வட்டம்

$\Rightarrow$

$$(x-\sqrt{2}-1)^2 + (y-1)^2 = 1^2$$

---

(10) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\tan^{-1}\left(\frac{1}{5}\right) = A$ என்க $\tan A = \frac{1}{5}$

$\tan^{-1}\left(\frac{5}{12}\right) = B$ $\tan B = \frac{5}{12}$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{\frac{2}{5}}{1-\frac{1}{25}}$$

$$= \frac{10}{24}$$

$$= \frac{5}{12} = \tan B$$

$\therefore 2A = B$

$$\tan(2B)$$

$$= \frac{2\tan B}{1-\tan^2 B}$$

$$= \frac{2\times 5/12}{1-(5/12)^2}$$

$$= \frac{120}{119}$$

Isuru

(11) a) $f(x) = ax^2 + 2x + c$ $\quad a \neq 0$ $\qquad g(x) = bx^2 + x + c$ $\quad b \neq 0$

$f(\alpha) = a\alpha^2 + 2\alpha + c = 0$ —— (1)

$g(\alpha) = b\alpha^2 + \alpha + c = 0$ —— (2)

(1) − (2) $\Rightarrow (a-b)\alpha^2 + \alpha = 0$

$\alpha = \dfrac{1}{b-a}$; $\alpha \neq 0$

(1) $\Rightarrow \dfrac{a}{(b-a)^2} + \dfrac{2}{(b-a)} + c = 0$

$\dfrac{a + 2b - 2a}{(b-a)^2} + c = 0$

$c = \dfrac{a-2b}{(b-a)^2}$

$f(x) = ax^2 + 2x + c = 0$

$\Delta_1 = 2^2 - 4ac$

$= 4\left(1 - \dfrac{(a-2b)a}{(b-a)^2}\right)$

$\Delta_1 = \dfrac{4b^2}{(b-a)^2}$ $\qquad b \neq 0 \therefore b^2 > 0$

$b \neq a$

$\therefore \Delta_1 > 0$

$$g(x) = bx^2 + x + c = 0$$

$$\Delta_2 = 1 - 4bc$$

$$= 1 - \frac{4b(a-2b)}{(b-a)^2}$$

$$= \frac{1}{(b-a)^2}\left[b^2 - 2ab + a^2 - 4ab + 8b^2 \right]$$

$$\Delta_2 = \frac{(a-3b)^2}{(b-a)^2}$$

$g(x)=0$ இற்கு பொருந்தும் மூலங்கள் உண்டு என்பதால் $\Delta_2 = 0$

$$(a-3b)^2 = 0$$

$$a = 3b \quad ; \; a \neq b$$

$(\beta) \Rightarrow$

$$\alpha + \beta = \frac{-2}{a}$$

$$\beta = \frac{-2}{a} - \frac{1}{b-a}$$

$$\beta = \frac{a - 2a + 2b}{a(a-b)}$$

$$\beta = \frac{a-2b}{a(b-a)}$$

$(\gamma) \Rightarrow$

$$\alpha + \gamma = \frac{-1}{b}$$

$$\gamma = \frac{-1}{b} + \frac{1}{a-b}$$

$$= \frac{b - a + b}{b(a-b)}$$

$$\gamma = \frac{a-2b}{b(b-a)}$$

(11) b) $P(x) = (x-1)\,g(x) + 7$

$x=1 \Rightarrow \quad P(1) = 7$

$P(x) = (x-3)\,h(x) + 13$

$x=3 \Rightarrow \quad P(3) = 13$

$P(x) = (x-1)(x-3)\,\phi(x) + Ax + B$ — (1)

$x=1 \Rightarrow \quad 7 = A + B.$

$x=3 \Rightarrow \quad 13 = 3A + B$

$A = 3$

$B = 4$

$P(x) = (x-1)(x-3)\,\phi(x) + 3x + 4$

$x=2 \Rightarrow \quad 6 = (1)(-1)\,\phi(2) + 6 + 4$

$\phi(2) = 4 = 2+2$

$\therefore \quad \phi(x) = (x+2)$

.

$P(x) = (x-1)(x-3)(x+2) + 3x + 4$

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12) a) i) $^6C_4\,^4C_2\,^2C_1 = \left(\frac{6!}{2!\,4!}\right)\left(\frac{4!}{2!\,2!}\right)\left(\frac{2!}{1.}\right)$

$= 180.$

ii) $^4C_2\,^2C_1\left(^4C_2 + {^4C_4}\right) = \left(\frac{4!}{2!\,2!}\right).2\left(\frac{4!}{2!\,2!} + 1\right)$

$= 84.$

iii) $^6C_4\,^4C_2\,^2C_1 - {^6C_4} \times 1 \times {^2C_1} = 180 - \left(\frac{6!}{4!\,2!} \times 2\right)$

$= 150.$

iv) $^6C_5\,^4C_3\,^2C_1 = \left(\frac{6!}{1!\,5!}\right)\left(\frac{4!}{3!}\right).2$

$= 6 \times 4 \times 2.$

$= 48.$

b) $u_r = f(r) - f(r+1)$

$\frac{1}{r(r+1)(r+2)} = \frac{\lambda}{r(r+1)} - \frac{\lambda}{(r+1)(r+2)}$

$1 = \lambda(r+2) - \lambda r$

$= \lambda r + 2\lambda - \lambda r$

$2\lambda = 1$

$\lambda = 1/2$ // $\therefore f_r = \frac{1}{2r(r+1)}$

$r=1 \Rightarrow u_1 = f(1) - f(2)$

$r=2 \Rightarrow u_2 = f(2) - f(3)$

$r=3 \Rightarrow u_3 = f(3) - f(4)$

$\vdots \qquad \vdots$

$r=n-1 \Rightarrow u_{n-1} = f(n-1) - f(n)$

$r=n \Rightarrow u_n = f(n) - f(n+1)$

$$\sum_{r=1}^{n} u_r = f(1) - f(n+1)$$

$$= \frac{1}{2(2)} - \frac{1}{2(n+1)(n+2)}$$

$$= \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

$r \longrightarrow r+1$

$$u_{r+1} = \frac{1}{(r+1)(r+2)(r+3)} = V_r.$$

$$\text{so, } V_r = f(r+1) - f(r+2)$$

$$\sum_{r=1}^{n} V_r = f(2) - f(n+2)$$

$$= \frac{1}{2\times2\times3} - \frac{1}{2\times(n+2)(n+3)}$$

$$\sum_{r=1}^{n} V_r = \frac{1}{12} - \frac{1}{2(n+2)(n+3)}$$

$$U_r + V_r = \frac{1}{r(r+1)(r+2)} + \frac{1}{(r+1)(r+2)(r+3)}$$

$$= \frac{r+3+r}{r(r+1)(r+2)(r+3)}$$

$$= \frac{2r+3}{r(r+1)(r+2)(r+3)} = W_r.$$

$$W_r = U_r + V_r.$$

$$\sum_{r=1}^{n} W_r = \sum_{r=1}^{n} U_r + \sum_{r=1}^{n} V_r.$$

$$= \frac{1}{4} - \frac{1}{2(n+1)(n+2)} + \frac{1}{12} - \frac{1}{2(n+2)(n+3)}$$

$$= \frac{1}{3} - \frac{1}{2(n+1)(n+2)} - \frac{1}{2(n+2)(n+3)}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} W_r = \lim_{n\to\infty}\left[\frac{1}{3} - \frac{1}{2(n+1)(n+2)} - \frac{1}{2(n+2)(n+3)}\right]$$

$$= \frac{1}{3}.$$

∴ எனவே, இத்தொடர் ஒருங்கும்.

13) (a) $A^T = \begin{pmatrix} a & 2 & b \\ 0 & -2 & 3 \end{pmatrix}$

$$A^T.B = \begin{pmatrix} a & 2 & b \\ 0 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 4 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} a+2b+8 & 5a+b-2 \\ -2 & 5 \end{pmatrix}$$

$a + 2b + 8 = 15$

$a + 2b = 7$ —— ①

$5a + b - 2 = 6$

$5a + b = 8$ —— ②

$c = -2$.

①, ② $\Rightarrow$ $a = 1$, $b = 3$

$$c = \begin{pmatrix} 15 & 6 \\ -2 & 5 \end{pmatrix}$$

$$c^{-1} = \frac{1}{87}\begin{pmatrix} 5 & -6 \\ 2 & 15 \end{pmatrix}$$

$c(P + 2I) = 3c + I$

$c^{-1}c(P + 2I) = 3c^{-1}c + c^{-1}I$

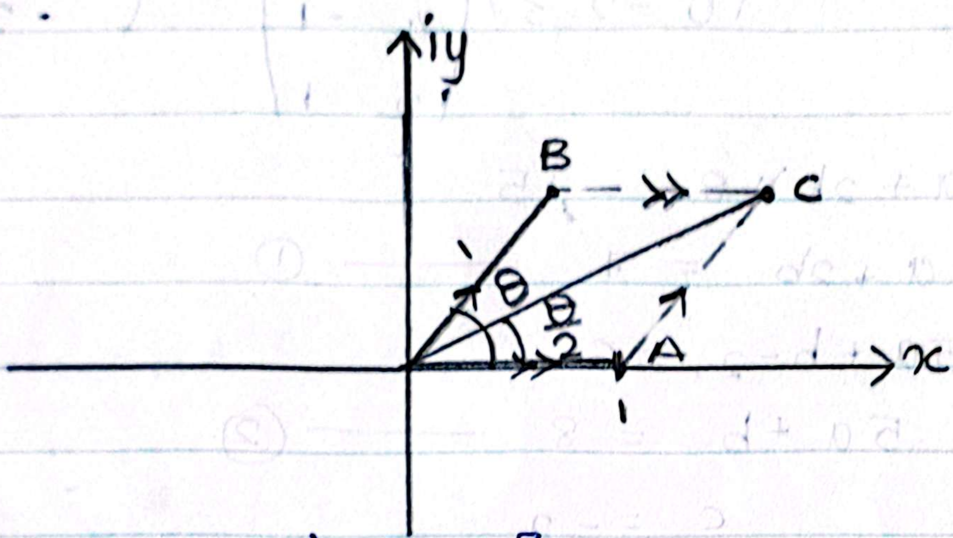
$P + 2I = 3I + c^{-1}$

$P = I + c^{-1}$

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{87}\begin{pmatrix} 5 & -6 \\ 2 & 15 \end{pmatrix}$$

$$= \frac{1}{87}\begin{pmatrix} 92 & -6 \\ 2 & 102 \end{pmatrix}_{2\times 2}$$

(b)



$\arg(z_1+z_2) = \frac{\theta}{2}$ [சாய்சதுரத்தின் மூலைவிட்டம் கோணத்தை இருசமகூறிடும்]

$|z_1+z_2| = 2\cos\left(\frac{\theta}{2}\right).$

$|z_1+z_2|\Big|_{max} = \left(2\cos\frac{\theta}{2}\right)\Big|_{max} = 2\cos\left(\frac{\theta}{2}\right)\Big|_{max}$

$\cos\left(\frac{\theta}{2}\right)\Big|_{max} = 1 \Rightarrow \frac{\theta}{2} = 0 \Rightarrow \theta = 0$

$|z_1+z_2|\Big|_{max} = 2$

$\Rightarrow z_2 = \cos 0 + i\sin 0 = 1$

$$|z_1+z_2|\Big|_{min} = \left(2\cos\frac{\theta}{2}\right)\Big|_{min} = 2\cos\left(\frac{\theta}{2}\right)\Big|_{min}$$

$$\cos\left(\frac{\theta}{2}\right)\Big|_{min} = 0 \Rightarrow \frac{\theta}{2} = \frac{\pi}{2} \Rightarrow \theta = \pi.$$

$$|z_1+z_2|\Big|_{min} = 0$$

$$\Rightarrow z_2 = \cos\pi + i\sin\pi$$
$$= -1$$

$$\frac{1}{z_1+z_2} = \frac{1}{2\cos\left(\frac{\theta}{2}\right)\left[\cos\left(\frac{\theta}{2}\right)+i\sin\left(\frac{\theta}{2}\right)\right]}$$

$$= \frac{\left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right)}{2\cos\left(\frac{\theta}{2}\right)}$$

$$= \left[\cos\left(-\frac{\theta}{2}\right)+i\sin\left(-\frac{\theta}{2}\right)\right] : 2\cos\left(\frac{\theta}{2}\right).$$

$$Re\left(\frac{1}{z_1+z_2}\right) = \frac{\cos(-\theta/2)}{2\cos(\theta/2)} \; ; \cos\frac{\theta}{2} = \cos\left(-\frac{\theta}{2}\right)$$

$$= \frac{1}{2}.$$

(c) $\bar{z} = r(\cos\alpha - i\sin\alpha)$

$= r[\cos(-\alpha) + i\sin(-\alpha)]$

$z^n + \bar{z}^n = [r(\cos\alpha + i\sin\alpha)]^n + [r(\cos -\alpha + i\sin -\alpha)]^n$

$= r^n(\cos n\alpha + i\sin n\alpha) + r^n(\cos(-n\alpha) + i\sin(-n\alpha))$

$= r^n(\cos n\alpha + i\sin n\alpha + \cos n\alpha - i\sin n\alpha)$.

$= 2r^n\cos(n\alpha)$.

$z_1 = 1 + i$

$\bar{z}_1 = 1 - i$

$z_1 = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$z_1^n + \bar{z}_1^n = 2(\sqrt{2})^n \cos\left(\frac{n\pi}{4}\right)$; $r = \sqrt{2}$, $\alpha = \frac{\pi}{4}$.

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(14) a) $$y = f(x) = \frac{2x(2x-1)(2x-5)}{(x-1)^3}$$

$$y = \frac{2x(4x^2-12x+5)}{(x-1)^3}$$

$$\left(\frac{dy}{dx}\right) = \frac{(x-1)^3(24x^2-48x+10) - (8x^3-24x^2+10x)3(x-1)^2}{(x-1)^6}$$

$$f'(x) = \frac{2(14x-5)}{(x-1)^4}$$

$$f''(x) = \frac{(x-1)^4(28) - (28x-10)4(x-1)^3}{(x-1)^8}$$

$$= \frac{12-12\times 7x}{(x-1)^5}$$

$$= \frac{12(1-7x)}{(x-1)^5} = \frac{12(1-\lambda x)}{(x-1)^5}$$

$\lambda = 7$

★ $(0,0)$, $(1/2, 0)$, $(5/2, 0)$ இல் அச்சை வெட்டும்

★ $x = 1$ ஆக $y \to \alpha$

$\therefore$ $x = 1$ நிலைக்குத்து அணுகுகோடு.

★ $$\lim_{x \to \pm\alpha} y = \lim_{x \to \pm\alpha} \frac{2x(2x-1)(2x-5)}{(x-1)^3} = \lim_{x \to \pm\alpha} \frac{2\left(2-\frac{1}{x}\right)\left(2-\frac{5}{x}\right)}{\left(1-\frac{1}{x}\right)^3}$$

$$= 2(2)(2) = 8$$

$x \to \pm\alpha$ ஆக $y \to 8$

$\therefore$ $y = 8$ கிடை அணுகுகோடு.

★ $f'(x) = 0$ இல் திரும்பல் புள்ளி பெறப்படும், $x = 5/14$ இல் திரும்பல் புள்ளி உண்டு.

	$-\alpha < x < 5/14$	$5/14 < x < 1$	$1 < x < +\alpha$
$f'(x)$ இன் குறி	$(-)$	$(+)$	$(+)$
$f(x)$ இன் தன்மை	$\searrow$	$\nearrow$	$\nearrow$

$x = 5/14$ இல் இழிவு புள்ளி

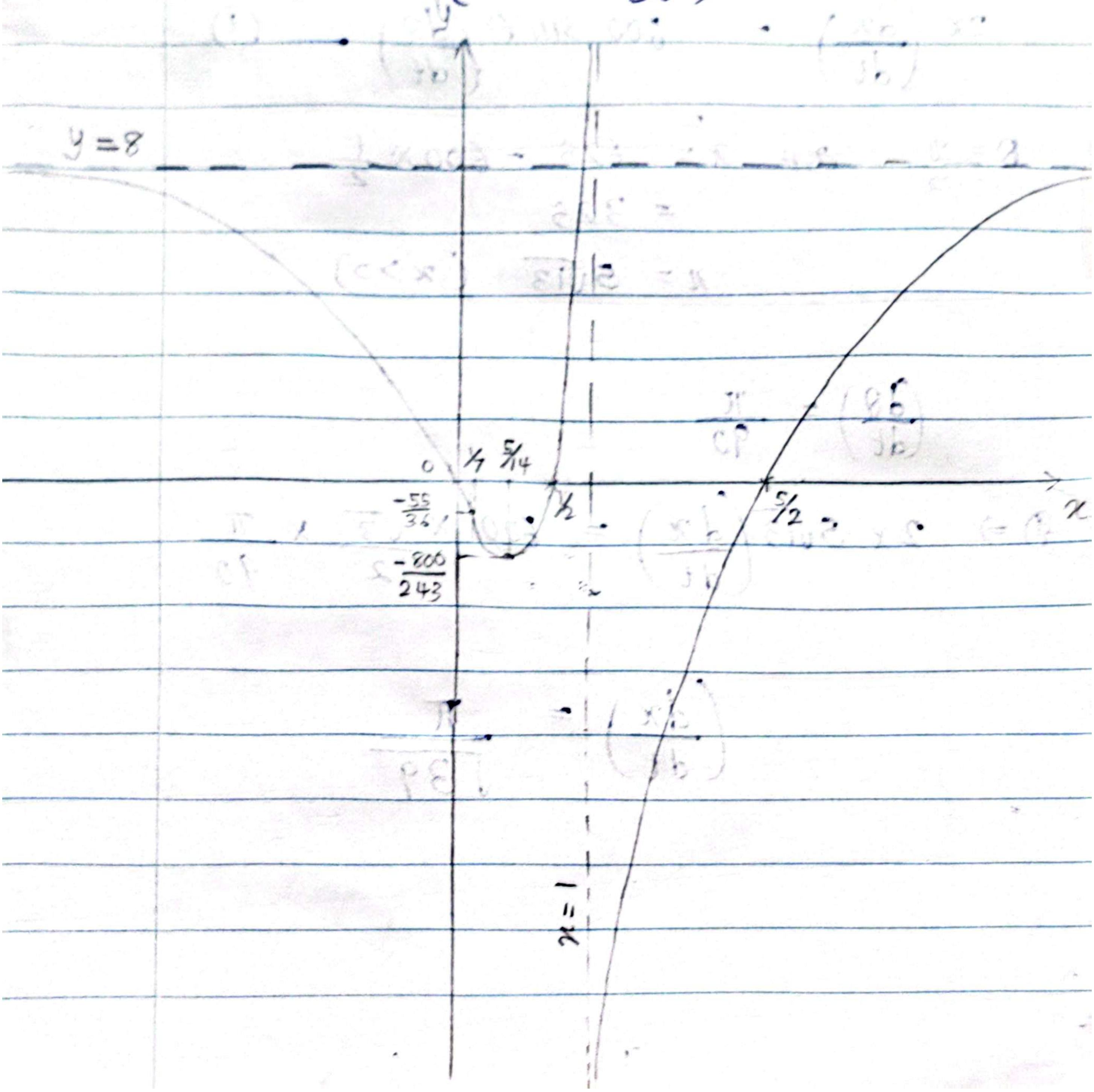
இழிவு புள்ளி $\equiv \left(5/14, \frac{-800}{243}\right)$

$f''(x) = 0$ ஆக விபத்திப்புள்ளி பெறுவோம்

$x = 1/7$ இல் விபத்தி உண்டு.

	$-\alpha < x < 1/7$	$1/7 < x < 1$	$1 < x < +\alpha$
$f''(x)$ இன் குறி	$(-)$	$(+)$	$(-)$
$f'(x)$ இன் குழிவுத் தன்மை	கீழ் நோக்கி	மேல் நோக்கி	கீழ் நோக்கி

$$\text{விபத்திப்புள்ளி} \equiv \left(1/7, \frac{-55}{36}\right)$$



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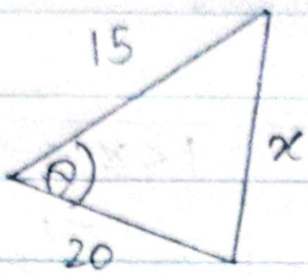
(14) b)



cos rule,

$$x^2 = 225 + 400 - 600\cos\theta$$

$$x^2 = 625 - 600\cos\theta$$

$$2x\left(\frac{dx}{dt}\right) = 600\sin\theta\left(\frac{d\theta}{dt}\right) \quad \text{— (1)}$$

$\theta = \frac{\pi}{3}$ විට $x^2 = 625 - 600 \times \frac{1}{2}$

$$= 325$$

$$x = 5\sqrt{13} \quad (x > 0)$$

$$\left(\frac{d\theta}{dt}\right) = \frac{\pi}{90}$$

$$(1) \Rightarrow \quad 2 \times 5\sqrt{13}\left(\frac{dx}{dt}\right) = 600 \times \frac{\sqrt{3}}{2} \times \frac{\pi}{90}$$

$$\left(\frac{dx}{dt}\right) = \frac{\pi}{\sqrt{39}}$$

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(15) a) $J = \int_0^{\pi/3} \tan^2\theta \sec\theta \, d\theta$

$= \int_0^{\pi/3} (\sec^2\theta - 1) \sec\theta \, d\theta = \int_0^{\pi/3} \sec^3\theta - \sec\theta \, d\theta$

$= \int_0^{\pi/3} \left(\frac{d\tan\theta}{d\theta}\right) \sec\theta \, d\theta - \int_0^{\pi/3} \sec\theta \, d\theta$

$= \left[\tan\theta \sec\theta\right]_0^{\pi/3} - \int_0^{\pi/3} \tan\theta \left(\frac{d\sec\theta}{d\theta}\right) d\theta - \int_0^{\pi/3} \sec\theta \, d\theta$

$= \left[\tan\theta \sec\theta\right]_0^{\pi/3} - \int_0^{\pi/3} \tan^2\theta \sec\theta \, d\theta - \int_0^{\pi/3} \sec\theta \, d\theta$

$2J = \left[\tan\theta\sec\theta - \ln|\sec\theta + \tan\theta|\right]_0^{\pi/3}$

$= \sqrt{3} \times 2 - \ln(2+\sqrt{3}) - 0$

$J = \sqrt{3} - \frac{1}{2}\ln(2+\sqrt{3})$

$\underline{I = \int_0^3 \ln(\sqrt{x+1} + \sqrt{x}) \, dx}$

$x^{1/2} = \tan\theta$ $\qquad x = 0 \Rightarrow \tan\theta = 0$

$\theta = 0$

$\frac{1}{2\sqrt{x}} = \sec^2\theta \left(\frac{d\theta}{dx}\right)$ $\qquad x = 3 \Rightarrow \theta = \pi/3$

$dx = 2\sqrt{x}(1+x) \, d\theta$

$dx = 2\tan\theta \sec^2\theta \, d\theta$

$$I = \int_0^{\pi/3} 2\tan\theta \sec^2\theta \ln(\sec\theta + \tan\theta)\, d\theta$$

$$= \int_0^{\pi/3} 2\tan\theta \left(\frac{d\tan\theta}{d\theta}\right) \ln(\sec\theta + \tan\theta)\, d\theta$$

$$= \left[2\tan^2\theta \ln(\sec\theta + \tan\theta)\right]_0^{\pi/3} - \int_0^{\pi/3} 2\tan\theta \left(\frac{d\,\tan\theta \ln(\sec\theta + \tan\theta)}{d\theta}\right) d\theta$$

$$= 6\ln(2+\sqrt{3}) - \int_0^{\pi/3} \frac{2\tan^2\theta \cancel{(\sec\theta + \tan\theta)}\sec\theta}{\cancel{(\sec\theta + \tan\theta)}} + 2\tan\theta\sec^2\theta \ln(\sec\theta + \tan\theta)\, d\theta$$

$$= 6\ln(2+\sqrt{3}) - 2J - I$$

$$2I = 6\ln(2+\sqrt{3}) - 2J$$

$$I = 3\ln(2+\sqrt{3}) - J$$

$$I = 3\ln(2+\sqrt{3}) - \sqrt{3} + \frac{1}{2}\ln(2+\sqrt{3})$$

$$I = \frac{1}{2}\left[7\ln(2+\sqrt{3}) - 2\sqrt{3}\right]$$

(15) b) $\dfrac{4}{(x-1)(x+1)^2} = \dfrac{A}{(x-1)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$

$$4 = A(x+1)^2 + B(x+1)(x-1) + C(x-1)$$

$x = 1 \Rightarrow A = 1$

$x = (-1) \Rightarrow C = (-2)$

$x = 0 \Rightarrow B = (-1)$

$$\frac{4}{(x-1)(x+1)^2} = \frac{1}{(x-1)} + \frac{(-1)}{(x+1)} + \frac{(-2)}{(x+1)^2}$$

$$\int \frac{4}{(x-1)(x+1)^2}\,dx = \int \frac{1}{(x-1)} + \frac{(-1)}{(x+1)} + \frac{-2}{(x+1)^2}\,dx$$

$$= \ln|x-1| - \ln|x+1| + \frac{2}{(x+1)} + c$$

தொகையீட்டு மாறிலி

$x = e^x$ என பிரதியிடுக.

$$\int \frac{4}{(e^x-1)(e^x+1)^2}\,de^x = \ln|e^x-1| - \ln|e^x+1| + \frac{2}{(e^x+1)} + c$$

$$\int \frac{4e^x}{(e^x-1)(e^x+1)^2}\,dx =$$

$$4\int \frac{1}{(1-e^{-x})(e^x+1)^2}\,dx =$$

$$\int \frac{1}{(1-e^{-x})(e^x+1)^2}\,dx = \frac{1}{4}\left(\ln|e^x-1| - \ln|e^x+1| + \frac{2}{e^x+1} + c\right)$$

15 c) $a+b-x=y$ எனின் ; $x \to a, \; y \to b$

$x \to b, \; y \to a$

$-1 = \left(\frac{dy}{dx}\right)$

$dy = -dx$

$$\int_a^b f(a+b-x)\,dx = \int_a^b f(y)\,dx$$

$$= \int_b^a f(y)(-1)\,dy$$

$$= \int_a^b f(y)\,dy$$

$$= \int_a^b f(x)\,dx$$

$$I = \int_1^3 \frac{\cos^2\left(\frac{\pi x}{8}\right)}{x(4-x)}\,dx = \int_1^3 \frac{\cos^2\left(\frac{\pi}{2} - \frac{\pi x}{8}\right)}{x(4-x)}\,dx$$

$$= \int_1^3 \frac{\sin^2\left(\frac{\pi x}{8}\right)}{x(4-x)}\,dx$$

$$= \int_1^3 \frac{1 - \cos^2\left(\frac{\pi x}{8}\right)}{x(4-x)}\,dx$$

$$I = \int_1^3 \frac{1}{x(4-x)}\,dx - \int_1^3 \frac{\cos^2\left(\frac{\pi x}{6}\right)}{x(4-x)}\,dx$$

$$I = \int_1^3 \frac{(1/4)}{x} + \frac{(1/4)}{(4-x)}\,dx - I$$

$$2I = \frac{1}{4}\Big[\ln|x| - \ln|4-x|\Big]_1^3$$

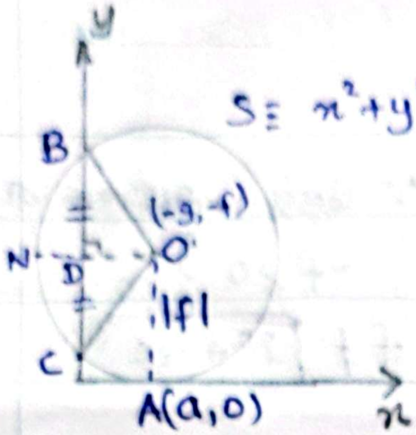
$$I = \frac{1}{8}\Big[(\ln 3 - \ln 1) - (\ln 1 - \ln 3)\Big]$$

$$= \frac{1}{8}\Big[\ln 3 + \ln 3\Big]$$

$$I = \frac{1}{4}\ln 3$$

(16)

a)



$S \equiv x^2+y^2+2gx+2fy+c=0$

x - அச்சைத் தொடுகையின்

$OA = r$

$|f| = \sqrt{f^2+g^2-c}$

$f^2 = f^2+g^2-c$

$g^2 = c$ ——— (1)

$OD = |g|$ $\quad ON = r = \sqrt{f^2+g^2-c}$

$f^2 > c$ ஆயின்

$f^2-c>0$

$f^2+g^2-c > g^2$

$\sqrt{f^2+g^2-c} > |g|$

$ON > OD$ $\therefore$ y - அச்சை வெட்டும்.

$BD^2 = BO^2 - OD^2$

$= (\sqrt{f^2+g^2-c})^2 - |g|^2$

$= f^2+g^2-c-g^2$

$= f^2-c$

$BD = \sqrt{f^2-c}$

நாண் $BC = 2\sqrt{f^2-c}$

$BC = l$ ஆயின், $l = 2\sqrt{f^2-c}$

$l^2/4 = f^2-c$

$(-f)^2 = \frac{l^2+4c}{4}$

$$-f = \frac{\sqrt{l^2+4c}}{2}$$ (∵ நேர் y-அச்சை வெட்டுவதால் $-f>0$)

$$f \neq \frac{\sqrt{l^2+4c}}{2}$$

$-g = a$

$g^2 = a^2$

$c = a^2$ (∵ ①ன் படி)

ஆரம் $= |f| = \frac{\sqrt{l^2+4c}}{2}$

வட்டத்தின் சமன்பாடு

$$(x+g)^2 + (y+f)^2 = r^2$$

$$(x-a)^2 + \left[y - \frac{\sqrt{l^2+4c}}{2}\right]^2 = \left(\frac{l^2+4c}{4}\right)$$

$c = a^2 \Rightarrow$

$$(x-a)^2 + \left[y - \frac{\sqrt{l^2+4a^2}}{2}\right]^2 = \left(\frac{l^2+4a^2}{4}\right)$$

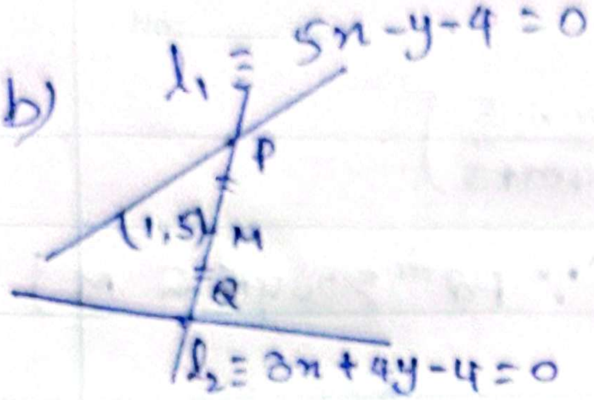
ΔABC பரப்பு $= \Delta OBC$ பரப்பு (∵ $BC \parallel OA$)

$= \frac{1}{2}(BC)(OD)$

$= \frac{1}{2}\left(\frac{l}{2}\right)(a)$

$= \frac{1}{2} \times \frac{10}{2} \times 12$

$= 30$ சதுர அலகு.

b) 

$P \equiv (\alpha, 5\alpha-4)$

$Q \equiv \left(\beta, \frac{4-3\beta}{4}\right)$ ଓମ୍ପେ

$$m = \frac{(5\alpha-4)-5}{\alpha-1} = \left(\frac{5\alpha-9}{\alpha-1}\right) \quad \text{—— ①}$$

$$\alpha m - m = 5\alpha - 9$$

$$\alpha = \frac{9-m}{5-m} \quad \text{—— } R_1$$

$$5\alpha - 4 = \frac{45-5m-20+4m}{5-m}$$

$$= \frac{25-m}{5-m}$$

$$\therefore P \equiv \left(\frac{9-m}{5-m}, \frac{25-m}{5-m}\right)$$

$$m = \frac{\left(\frac{4-3\beta}{4}\right)-5}{\beta-1} = \frac{-3\beta-16}{4(\beta-1)} \quad \text{—— ②}$$

$$4m\beta - 4m = -3\beta - 16$$

$$\beta = \frac{4m-16}{4m+3} \quad \text{—— } R_2$$

$$\frac{4-3\beta}{4} = 1 - \frac{3m-12}{4m+3}$$

$$= \frac{m+15}{}$$

$$\therefore Q \equiv \left(\frac{4m-16}{4m+3} , \frac{m+15}{4m+3} \right)$$

$$\frac{\alpha+\beta}{2} = 1$$ (∵ PQ ன் நடுப்புள்ளி M)

$$\alpha+\beta = 2$$

$$\frac{9-m}{5-m} + \frac{4m-16}{4m+3} = 2$$

$$(4m+3)(9-m) + (4m-16)(5-m) = 2(5-m)(4m+3)$$

$$35m = 83$$

$$m = 83/35$$

PQ ன் சமன்பாடு

$$\frac{y-5}{x-1} = 83/35$$

$$35y - 175 = 83x - 83$$

$$83x - 35y + 92 = 0$$

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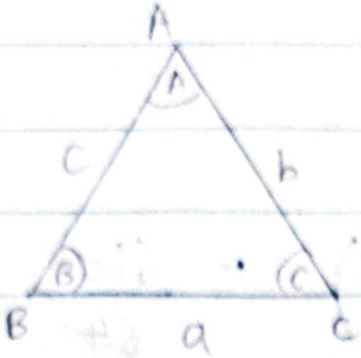
17) a)



sin rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = K$$ எனி

cos rule,

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$L.H.S = a\cos A + b\cos B$$

$$= K\sin A\cos A + K\sin B\cos B$$

$$= \frac{K}{2}(\sin 2A + \sin 2B)$$

$$= \frac{K}{\cancel{2}}(\cancel{2}\sin(A+B)\cos(A-B))$$

$$= K\sin(\pi - C)\ \frac{61}{64}$$

$$= K\sin C\left(\frac{61}{64}\right) = c\left(\frac{61}{64}\right) = R.H.S$$

$$a\cos A + b\cos B = c\left(\frac{61}{64}\right) \quad \text{— ①}$$

cos rule, $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos A = \left(\frac{5 + c^2}{6c}\right) \text{— } R_1 \qquad \cos B = \left(\frac{c^2 - 5}{4c}\right) \text{— } R_2$$

① , R1 , R2 $\Rightarrow$

$$2\left(\frac{5+c^2}{6c}\right) + 3\left(\frac{c^2-5}{4c}\right) = \frac{61c}{64}$$

$$\frac{320+64c^2}{3} + 48c^2 - 240 = 61c^2$$

$$25c^2 = 400$$

$$c^2 = 16$$

$$c = \pm 4$$

$$c = 4 \quad (c>0)$$

b) $t = \tan(\theta/2)$

$$\sin\theta = \frac{2\sin\theta/2\cos\theta/2}{\sin^2\theta/2+\cos^2\theta/2}$$

$$= \frac{2\tan\theta/2}{1+\tan^2\theta/2}$$

$$= \frac{2t}{1+t^2}$$

$$\cos\theta = \frac{\cos^2\theta/2-\sin^2\theta/2}{\cos^2\theta/2+\sin^2\theta/2} = \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} = \frac{1-t^2}{1+t^2}$$

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$$\frac{1+\sin\theta}{3+2\cos\theta} = \frac{1+\frac{2t}{1+t^2}}{3+\frac{2(1-t^2)}{1+t^2}}$$

$$= \frac{1+t^2+2t}{3+3t^2+2-2t^2}$$

$$= \frac{(1+t)^2}{5+t^2}$$

$$y = \frac{1+\sin\theta}{3+2\cos\theta} = \frac{(1+t)^2}{5+t^2}$$ என்க.

$$y(5+t^2) = 1+2t+t^2$$

$$0 = (1-y)t^2 + 2t + (1-5y)$$

t இன் மெய் பெறுமானங்களுக்கு

$$\Delta \geq 0$$

$$4 - 4(1-y)(1-5y) \geq 0$$

$$0 \geq y\left(y - 6/5\right)$$

$$0 \leq y \leq 6/5$$

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(17) c) $\cos x + \cos 2x + \cos 3x = \sin x + \sin 2x$

$2\cos 2x \cos x + \cos 2x = \sin x(1 + 2\cos x)$

$\cos 2x(2\cos x + 1) - \sin x(2\cos x + 1) = 0$

$(2\cos x + 1)(\cos 2x - \sin x) = 0$

case 1

$(2\cos x + 1) = 0$

$\cos x = \frac{-1}{2}$

$\cos x = \cos\left(\frac{2\pi}{3}\right)$

$x = 2n\pi \pm \frac{2\pi}{3}; n \in \mathbb{Z}$

case 2

$\cos 2x - \sin x = 0$

$1 - 2\sin^2 x - \sin x = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = (-1)$

$\sin x = \sin(\pi/6)$ $\quad$ $\sin x = \sin(3\pi/2)$

$x = n\pi + (-1)^n\left(\frac{\pi}{6}\right)$

$x = n\pi + (-1)^n\left(\frac{3\pi}{2}\right)$

$\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ $\quad$ $\tan\alpha = \left(\frac{5}{12}\right)$ $\quad$ $\cos\alpha = \frac{12}{13}$ $\quad$ $\sin\alpha = \frac{5}{13}$

$\beta = \tan^{-1}\left(\frac{3}{4}\right)$ $\quad$ $\tan\beta = \left(\frac{3}{4}\right)$ $\quad$ $\cos\beta = \frac{4}{5}$ $\quad$ $\sin\beta = \frac{3}{5}$

$\tan\alpha = \frac{5}{12}$ $\quad$ $\tan\beta = \frac{9}{12}$ $\quad$ $\therefore \beta > \alpha$

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$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= \left(\frac{12}{13}\right)\left(\frac{4}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$$

$$= \frac{48+15}{65}$$

$$= \frac{63}{65}.$$

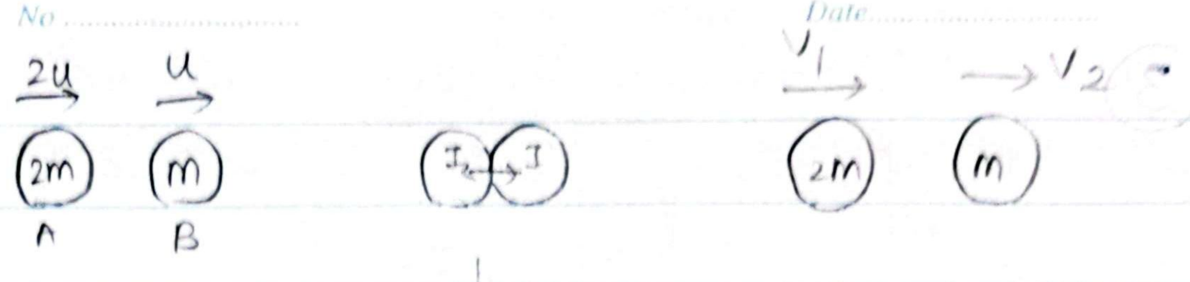
$$\sin^2(\alpha-\beta) = 1 - \cos^2(\alpha-\beta)$$

$$\sin(\alpha-\beta) = -\sqrt{1-\left(\frac{63}{65}\right)^2} \qquad (\sin(\alpha-\beta)<0)$$

$$\sin(\alpha-\beta) = -\sqrt{\frac{65^2-63^2}{65^2}}$$

$$= \frac{-1}{65}\left(\sqrt{256}\right)$$

$$\sin(\alpha-\beta) = \frac{-16}{65}$$

① 

தொகுதி → $I = \Delta mV$

$$0 = mV_2 + 2mV_1 - 5mu$$

$$5u = V_2 + 2V_1 \quad —①$$

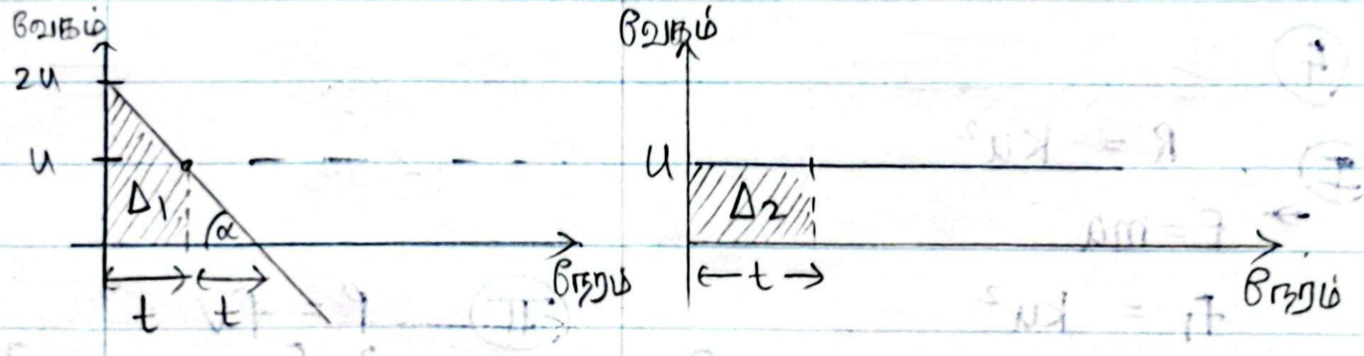
N·L·R ; $\dfrac{V_2 - V_1}{2u - u} = \dfrac{1}{2}$

$$\frac{u}{2} = V_2 - V_1 \quad —②$$

②, ① ⇒ $V_1 = \dfrac{3u}{2}$

→ $V_1 > 0$

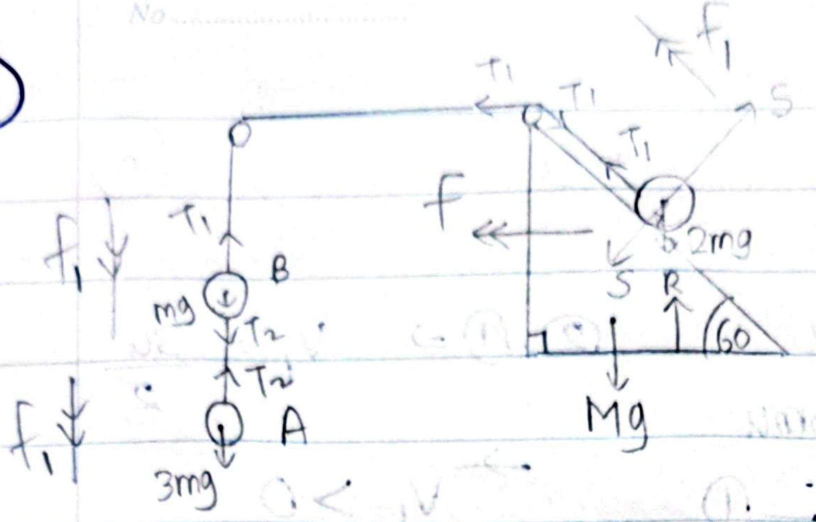
∴ A மற்றும் B கிற்கு இடையே மீண்டும் ஒரு மோதுகை நடைபெறும்



$$\tan\alpha = g = \frac{2u}{2t} \Rightarrow t = \frac{u}{g}$$

$$\frac{\Delta_2}{\Delta_1} = \frac{u \times (u/g)}{\frac{1}{2} \times (2u+u)(u/g)} = \frac{u}{\frac{1}{2} \times 3u} = \frac{2}{3}$$

③



(3m) $\downarrow F=ma$

$3mg - T_2 = 3m(f_1)$

(2m) $F = ma$

$T_1 - 2mg\sin 60 = 2m(f_1 + F\cos 60)$

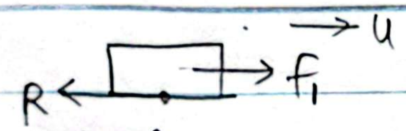
$(m+3m) \downarrow F = ma$

$4mg - T_1 = 4m(f_1)$

$(2m, M) \leftarrow F = ma$

$T_1 = MF + 2m(F + f_1\cos 60)$

④ I



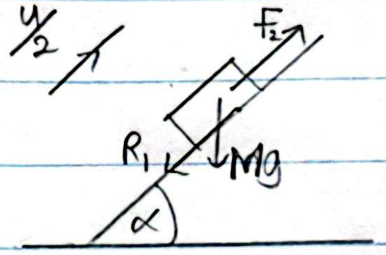
$R = ku^2$

I $\rightarrow F = ma$

$F_1 = ku^2$

$P = Fv = ku^2 \times u = ku^3$

II



II $R_1 = \dfrac{ku^2}{4}$

$F = ma$

$F_1 - \dfrac{ku^2}{4} - Mg\sin\alpha = 0$

$F_1 = \dfrac{ku^2}{4} + Mg\sin\alpha$

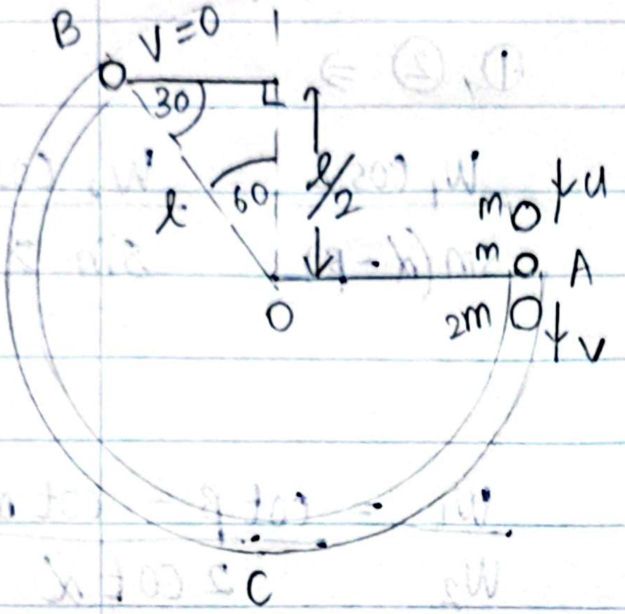
II $P = Fv$

$ku^3 = \left[\dfrac{ku^2}{4} + Mg\sin\alpha\right]\dfrac{u}{2}$

$2ku^2 = \dfrac{ku^2}{4} + Mg\sin\alpha$

$k = \dfrac{4Mg\sin\alpha}{7u^2}$

⑤



$2 \cdot$ கா ↓ $mu = 2m(V)$

$V = \frac{u}{2}$

பொறிமுறை சக்தி காப்பு விதி,

$$\frac{1}{2} \times 2m \left(\frac{u}{2}\right)^2 = (2m) g \left(\frac{l}{2}\right)$$

$$u^2 = 4gl$$

$$u = 2\sqrt{gl}$$

---

⑥ $\underline{a} = \underline{i} + 2\underline{j}$

$\underline{b} = 2\underline{i} - \underline{j}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= 2\underline{i} - \underline{j} - \underline{i} - 2\underline{j}$

$= \underline{i} - 3\underline{j}$

$\overrightarrow{CD} = (2\mu\underline{i} - \mu\underline{j}) - \lambda\underline{i} - 2\lambda\underline{j}$

$= (2\mu - \lambda)\underline{i} - (\mu + 2\lambda)\underline{j}$

$AB \perp CD$ ∴

$[(2\mu + \lambda)\underline{i} - (\mu + 2\lambda)\underline{j}](\underline{i} - 3\underline{j}) = 0$

$(2\mu - \lambda) + 3(\mu + 2\lambda) = 0$

$\lambda = -\mu$

$|\overrightarrow{CD}| = 2\sqrt{10}$

$40 = (2\mu - \lambda)^2 + (\mu + 2\lambda)^2$

$40 = 9\lambda^2 + \lambda^2$

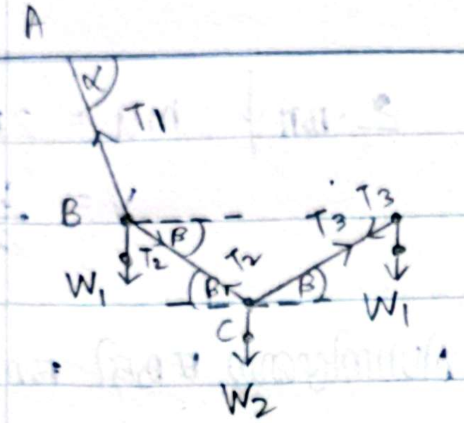
$\lambda^2 = 4$

$\lambda = \pm 2$

$\lambda = 2 \quad (\lambda > \mu)$

$\mu = (-2)$

(7)



①, ② ⇒

$$\frac{W_1 \cos\alpha}{\sin(\alpha-\beta)} = \frac{W_2 \cos\beta}{\sin 2\beta}$$

C, லாமியின் தேற்றம்,

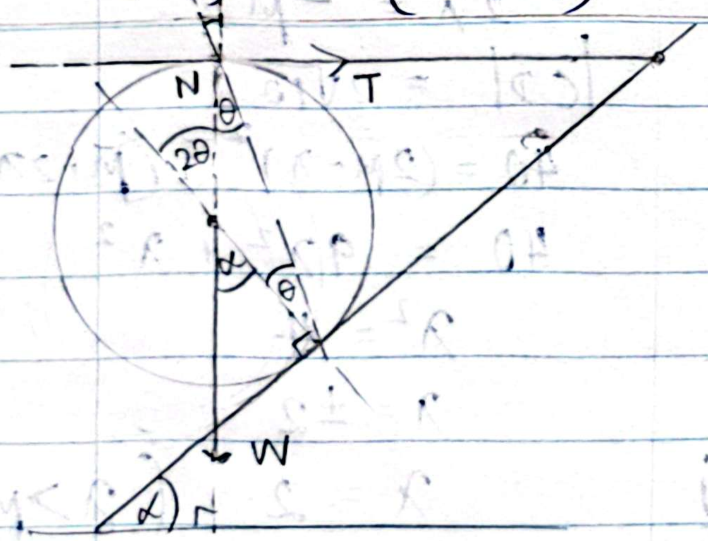
$$\frac{T_2}{\sin(90+\beta)} = \frac{W_2}{\sin(180-2\beta)} \quad \text{—①}$$

$$\frac{W_1}{W_2} = \frac{\cot\beta - \cot\alpha}{2\cot\alpha}$$

B, லாமியின் தேற்றம்,

$$\frac{T_2}{\sin(90+\alpha)} = \frac{W_1}{\sin(180-\alpha+\beta)} \quad \text{—②}$$

(8)



புள்ளி N இல்
லாமியின் தேற்றப்படி

$$\frac{W}{\sin(90+\alpha/2)} = \frac{T}{\sin(180-\alpha/2)}$$

$$T = W\tan(\alpha/2)$$

$2\theta = \alpha$

$\theta = \alpha/2$

கோணச் சமநிலைக்கு

$\theta \leq \lambda$

$\frac{\alpha}{2} \leq \lambda$

$\alpha \leq 2\lambda$

(9) $P(A \cap B') = 0.2$

$P(A' \cap B) = 0.15$

$P(A \cap B) = 0.1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.25 - 0.1$

$= 0.45.$

$P(A) = P(A \cap B') + P(A \cap B)$

$= 0.2 + 0.1$

$= 0.3$

$P(B) = P(A' \cap B) + P(A \cap B)$

$= 0.15 + 0.1$

$= 0.25$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.3} = \frac{1}{3}$$

(10) $y_i = x_i - 50$ எனின் $\bar{y} = \bar{x} - 50$, $S_y^2 = S_x^2$

$$\sum_{i=1}^{100} y = 57.2$$

$$\bar{y} = \frac{\sum_{i=1}^{100} y}{100} = 0.572$$

$\bar{x} = \bar{y} + 50$

$= 50.572$

$$S_y^2 = \frac{\sum_{i=1}^{100} y^2}{100} - (\bar{y})^2$$

$$S_x^2 = \frac{95.1}{100} - (0.572)^2$$

$= 0.951 - 0.327$

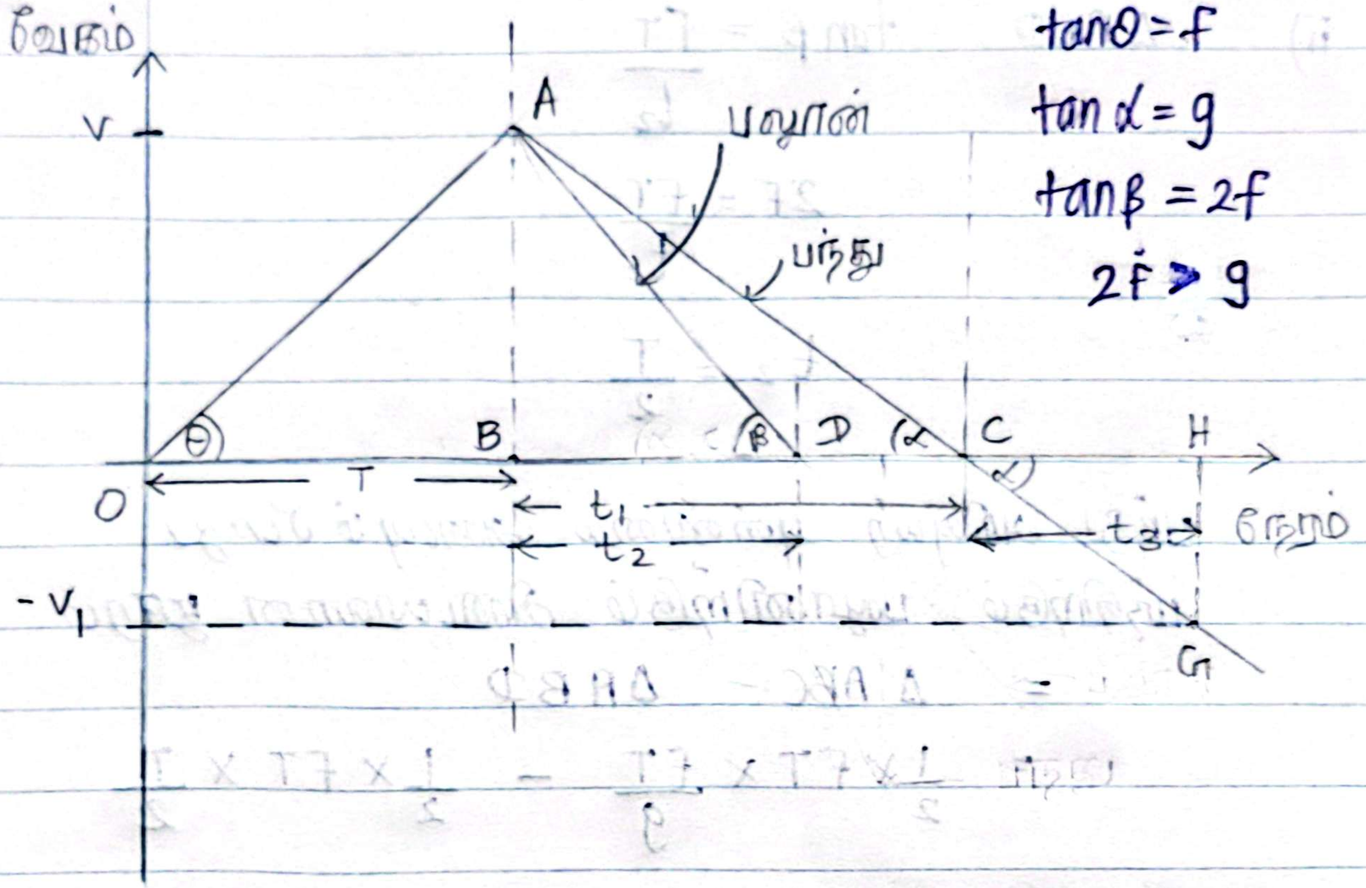
$= 0.624$

$\simeq 0.62$

$$\boxed{\bar{x} = 50.572}$$

$$\boxed{(S_x^2) = 0.62}$$

11 a)



$\tan\theta = f$

$\tan\alpha = g$

$\tan\beta = 2f$

$2f > g$

$\Delta OAB,\ \tan\theta = \dfrac{V}{T}$

$f = \dfrac{V}{T}$

$V = fT$

$\Delta ABC,\ \tan\alpha = \dfrac{V}{t_1}$

$\therefore g = \dfrac{fT}{t_1}$

$t_1 = \dfrac{fT}{g}$

i) ΔOAC இன் பரப்பு $= \dfrac{1}{2} \times fT(T + t_1)$

$= \dfrac{1}{2} \times fT\left(T + \dfrac{fT}{g}\right)$

$= \dfrac{fT^2}{2g}(f + g)$

ii) ΔABD, $\tan\beta = \dfrac{FT}{t_2}$

$$2F = \frac{FT}{t_2}$$

$$t_2 = \frac{T}{2}$$

பந்து அதியுயர் புள்ளியை அடையும் போது பந்திற்கும் பயணிற்கும் இடையிலான தூரம்

$$= \Delta ABC - \Delta ABD$$

$$= \frac{1}{2} \times FT \times \frac{FT}{g} - \frac{1}{2} \times FT \times \frac{T}{2}$$

$$= \frac{FT^2}{2}\left(\frac{F}{g} - \frac{1}{2}\right)$$

$$= \frac{FT^2}{4g}(2F - g)$$

நேரம் $= T + t_1 + t_3$ இல் பந்து மீண்டும் பாதையை அடைகிறது.

$$\Delta CHG, \quad \tan\alpha = \frac{v_1}{t_3}$$

$$g = \frac{v_1}{t_3}$$

$$v_1 = g t_3$$

$$\Delta CHG = \frac{fT^2}{4g}(2f-g) = \frac{1}{2} \times t_3 \times g t_3$$

$$t_3^2 = \frac{2fT^2(2f-g)}{4g^2}$$

$$t_3 = \frac{T}{2g}\sqrt{2f(2f-g)} \quad ; (t_3 > 0)$$

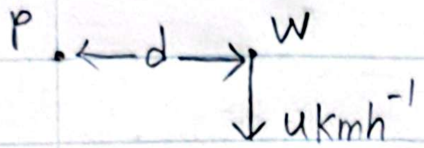
$$\text{நேரம்} = T + \frac{fT}{g} + \frac{T}{2g}\sqrt{2f(2f-g)}$$

$$= T\left(1 + \frac{f}{g} + \frac{\sqrt{2f(2f-g)}}{2g}\right)$$

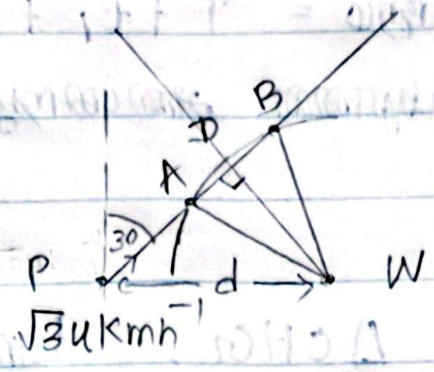
E → புவி , W – போர்க்கப்பல் , P – எதிரிகப்பல்

11 b)

புவி



W

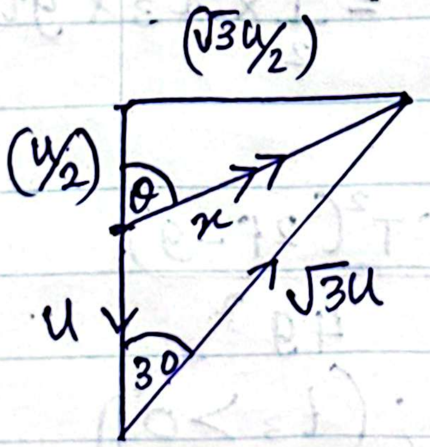


$V_{W,E} = \downarrow u\,kmh^{-1}$

$V_{P,W} = \nearrow^{30} \sqrt{3}u\,kmh^{-1}$

$V_{P,E} = V_{P,W} + V_{W,E}$

$= \nearrow^{30} \sqrt{3}u + \downarrow u$



$\tan\theta = \frac{\sqrt{3}u/2}{u/2} = \sqrt{3}$

$\therefore \theta = 60^\circ$

$x \sin 60 = \frac{\sqrt{3}u}{2}$

$x = u$

$V_{P,E} = \nearrow^{60} u$

ii) மிகக் குறுகிய தூரம் $= d\sin 60$

$$= \frac{\sqrt{3}d}{2}$$

iii) $AD^2 + \left(\frac{5\sqrt{3}}{10}d\right)^2 = \left(\frac{9}{10}d\right)^2$

$$AD^2 + \frac{75d^2}{100} = \frac{81d^2}{100}$$

$$AD^2 = \frac{6d^2}{100}$$

$$AD = \sqrt{6} \times \frac{d}{10} \quad ; (AD > 0)$$

எதிரிக்கப்பல் தாக்குதலுக்கு உட்படும் நேரம்

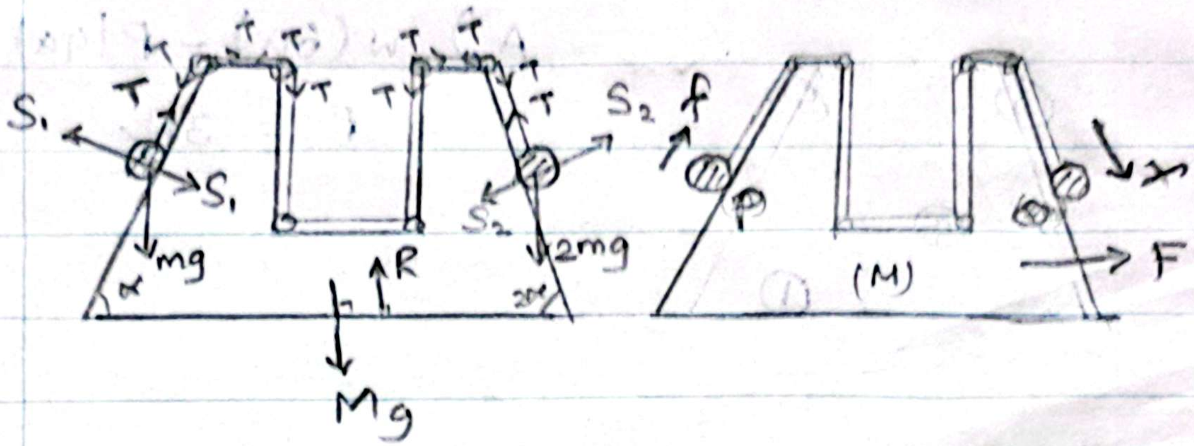
$$= \frac{\text{தூரம்}}{\text{வேகம்}}$$

$$= \frac{AB}{\sqrt{3}u} \times 60 \text{ mins}$$

$$= \frac{2\sqrt{2} \times \cancel{\sqrt{3}}d \times 60}{10 \times \cancel{\sqrt{3}}u}$$

$$= 12\sqrt{2}\,\frac{d}{u} \text{ minutes}$$

12) a)



$a_{ME} = \overrightarrow{F}$

$a_{PE} = f \underline{\nearrow \alpha} + \overrightarrow{F}$

$a_{QE} = \underline{2\alpha \swarrow} f + \overrightarrow{F}$

ஒதாகுதி $\rightarrow$ $F = ma$

$0 = MF + m(F + f\cos\alpha) + 2m(F + f\cos 2\alpha)$

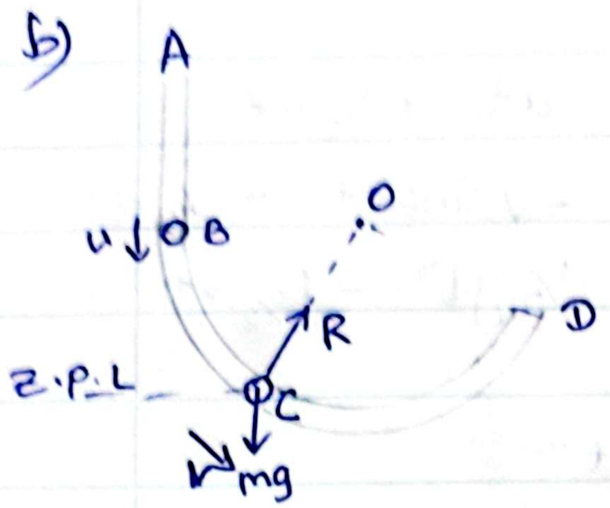
(P) $\underline{\nearrow \alpha}$ $F = ma$

$T - mg\sin\alpha = m(f + F\cos\alpha)$

(Q) $\underline{2\alpha \swarrow}$ $F = ma$

$2mg\sin 2\alpha - T = 2m(f + F\cos 2\alpha)$

b)



i) $v^2 = u^2 + 2gs \quad (A \longrightarrow B)$

$u^2 = 2ag$

$u = \sqrt{2ag}$

பொறிமுறை சக்திகாப்பு

$$mg(2a\sin\theta) + \tfrac{1}{2}mu^2 = \tfrac{1}{2}mv^2$$

$$v^2 = 4ag\sin\theta + 2ag$$

$$v = \sqrt{2ag(1 + 2\sin\theta)} \quad \text{——— (1)}$$

$\nearrow \quad F = ma$

$$R - mg\sin\theta = \frac{mv^2}{2a}$$

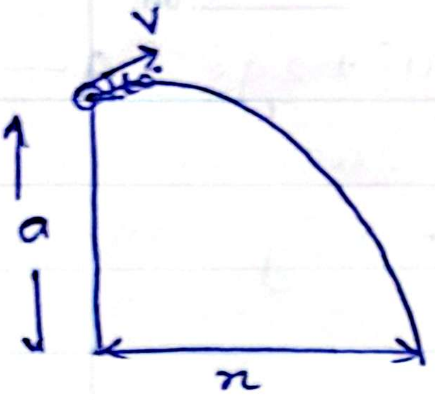
$$R = 3mg\sin\theta + mg$$

$$R = mg(1 + 3\sin\theta) \quad \text{——— (2)}$$

ii) (1) ல் $\theta = 150^\circ$ என பிரதியிட

$$v = \sqrt{4ag(\tfrac{1}{2}) + 2ag}$$

$$= 2\sqrt{ag}$$



$$\downarrow \quad S = ut + \tfrac{1}{2}at^2$$

$$a = -v\sin 60\, t + \tfrac{1}{2}gt^2$$

$$a = \tfrac{1}{2}gt^2 - \tfrac{\sqrt{3}}{2}vt \quad \text{—— } (3)$$

$$\rightarrow \quad S = ut + \tfrac{1}{2}at^2$$

$$x = vt/2 \quad \text{—— } (4)$$

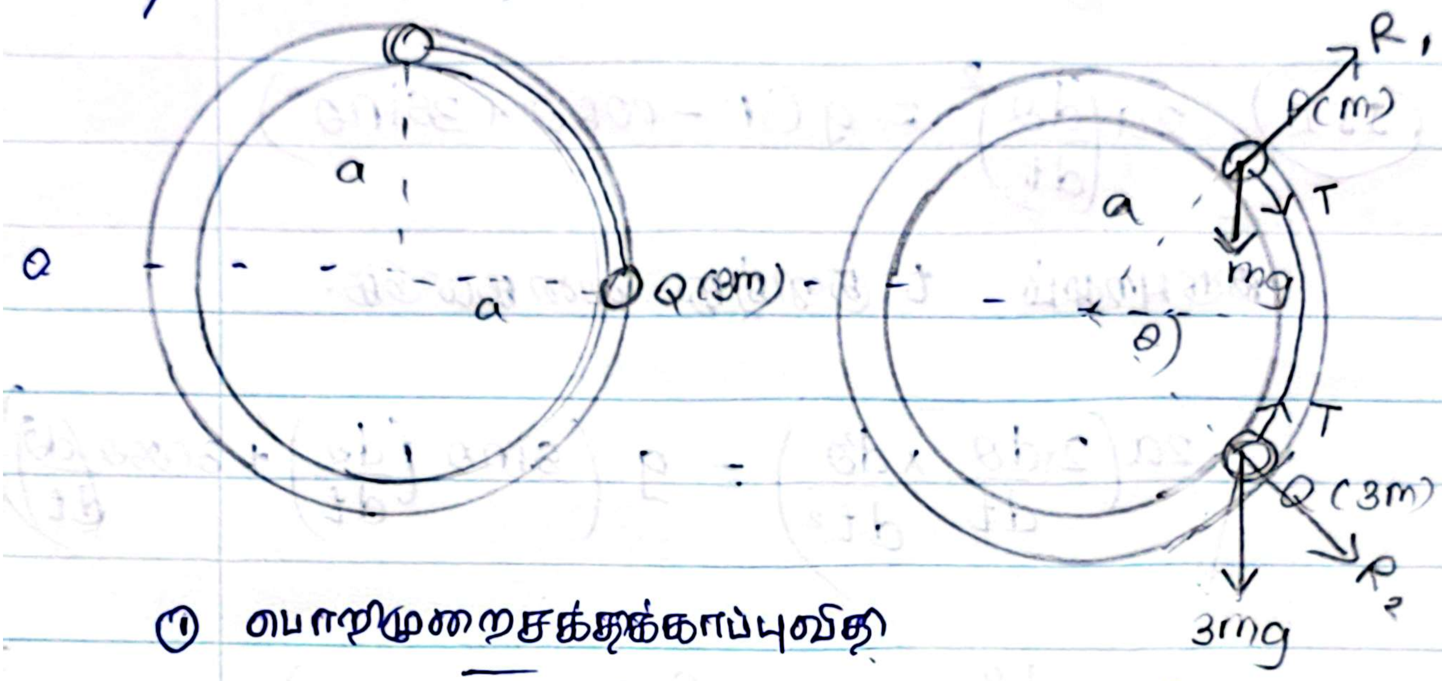
$$(3), (4) \Rightarrow \quad a = -\sqrt{3}x + \tfrac{1}{2}g\left(\frac{4x^2}{v^2}\right)$$

$$a = x^2/2a - \sqrt{3}x$$

$$2a^2 = x^2 - 2\sqrt{3}ax$$

$$x^2 - 2\sqrt{3}ax - 2a^2 = 0$$

13)



(i) பொறிமுறைசக்திக்காப்புவிதி

$$mga = mga\cos\theta - 3mga\sin\theta + \frac{1}{2}m\left(a\frac{d\theta}{dt}\right)^2 + \frac{1}{2}3m\left(a\frac{d\theta}{dt}\right)^2$$

$$2a\left(\frac{d\theta}{dt}\right)^2 = g(1 - \cos\theta + 3\sin\theta)$$

(II) $F = ma$

$$mg\cos\theta - R_1 = ma\left(\frac{d\theta}{dt}\right)^2$$

$$mg\cos\theta - R_1 = \frac{mg}{2}(1 - \cos\theta + 3\sin\theta)$$

$$R_1 = \frac{1}{2}mg(3\cos\theta - 1 - 3\sin\theta)$$

18

III $2a\left(\frac{d\theta}{dt}\right)^2 = g(1 - \cos\theta + 3\sin\theta)$

இருபுறமும் t குறித்து வகையிடுக.

$$2a\left(2 \times \frac{d\theta}{dt} \times \frac{d^2\theta}{dt^2}\right) = g\left(\sin\theta\left(\frac{d\theta}{dt}\right) + 3\cos\theta\left(\frac{d\theta}{dt}\right)\right)$$

$$4a\frac{d^2\theta}{dt^2} = g(\sin\theta + 3\cos\theta)$$

$$a\frac{d^2\theta}{dt^2} = \frac{1}{4}g(\sin\theta + 3\cos\theta)$$

IV $\searrow$ $F = ma$

$$T + mg\sin\theta = m\left(a\frac{d^2\theta}{dt^2}\right)$$

$$T + mg\sin\theta = m \times \frac{1}{4}g(\sin\theta + 3\cos\theta)$$

$$T = \frac{3}{4}mg(\cos\theta - \sin\theta)$$

(IV) $T = 0$ (இழை தொய்யும் போது)

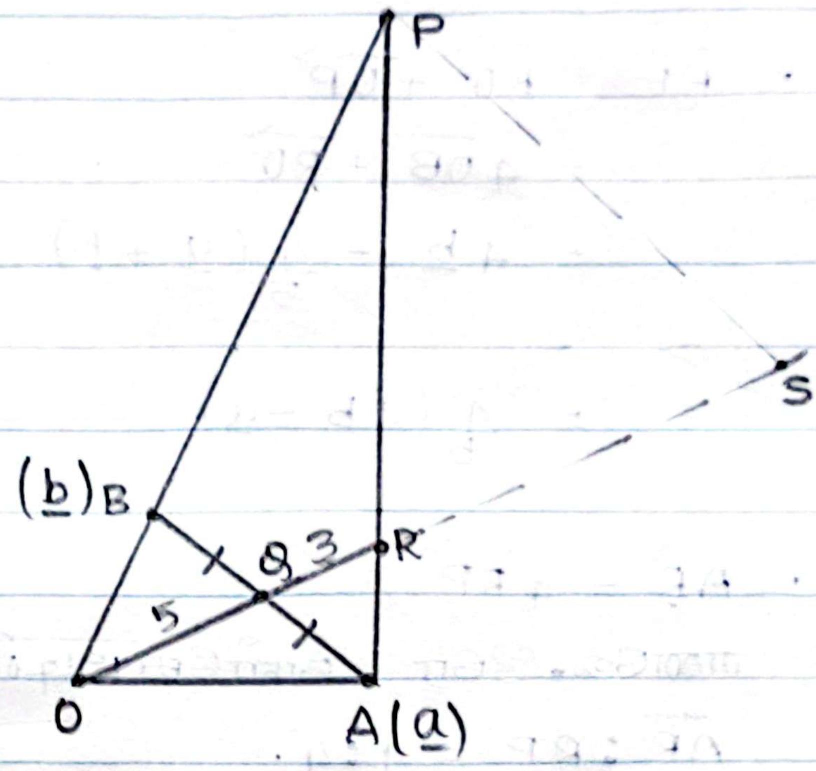
$$\cos\theta - \sin\theta = 0$$

$$\tan\theta = 1$$

$0 \leq \theta \leq \pi/2$ ஆகவே

$$\theta = \pi/4$$

14) (a)



- $\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$

$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \overrightarrow{OA} + \frac{1}{2}(\overrightarrow{AO} + \overrightarrow{OB})$

$= \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$

$= \frac{1}{2}(\underline{a} + \underline{b})$

- $\overrightarrow{OR} = \frac{8}{5}\overrightarrow{OQ}$

$= \frac{8}{5} \times \frac{1}{2}(\underline{a} + \underline{b})$

$= \frac{4}{5}(\underline{a} + \underline{b})$

- $\overrightarrow{AR} = \overrightarrow{AO} + \overrightarrow{OR}$

$= -\underline{a} + \frac{4}{5}(\underline{a} + \underline{b})$

$= \frac{1}{5}(+\underline{b} - \underline{a})$

- $\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OP}$

$$= 4\overrightarrow{OB} + \overrightarrow{RO}$$

$$= 4\underline{b} - \frac{4}{5}(\underline{a} + \underline{b})$$

$$= \frac{4}{5}(4\underline{b} - \underline{a})$$

- $\overrightarrow{AR} = 4\overrightarrow{RP}$

எனவே, ஒரே நேர்கோட்டில் இருக்கும்.

$AR : RP = 1:4$.

$$\overrightarrow{PS} = \lambda\overrightarrow{BA}$$

$$\overrightarrow{OS} = \mu\overrightarrow{OQ}$$

$$\overrightarrow{OP} + \overrightarrow{PS} = \overrightarrow{OS}$$

$$4\overrightarrow{OB} + \overrightarrow{PS} = \overrightarrow{OS}$$

$$4\underline{b} + \lambda\overrightarrow{BA} = \mu\overrightarrow{OQ}$$

$$4\underline{b} + \lambda(\overrightarrow{BO} + \overrightarrow{OA}) = \mu.\frac{1}{2}(\underline{a} + \underline{b})$$

$$4\underline{b} + \lambda(-\underline{b} + \underline{a}) = \frac{\mu}{2}(\underline{a} + \underline{b})$$

$\frac{\mu}{2} = \lambda$ $\qquad$ $\frac{\mu}{2} = 4 - \lambda$.

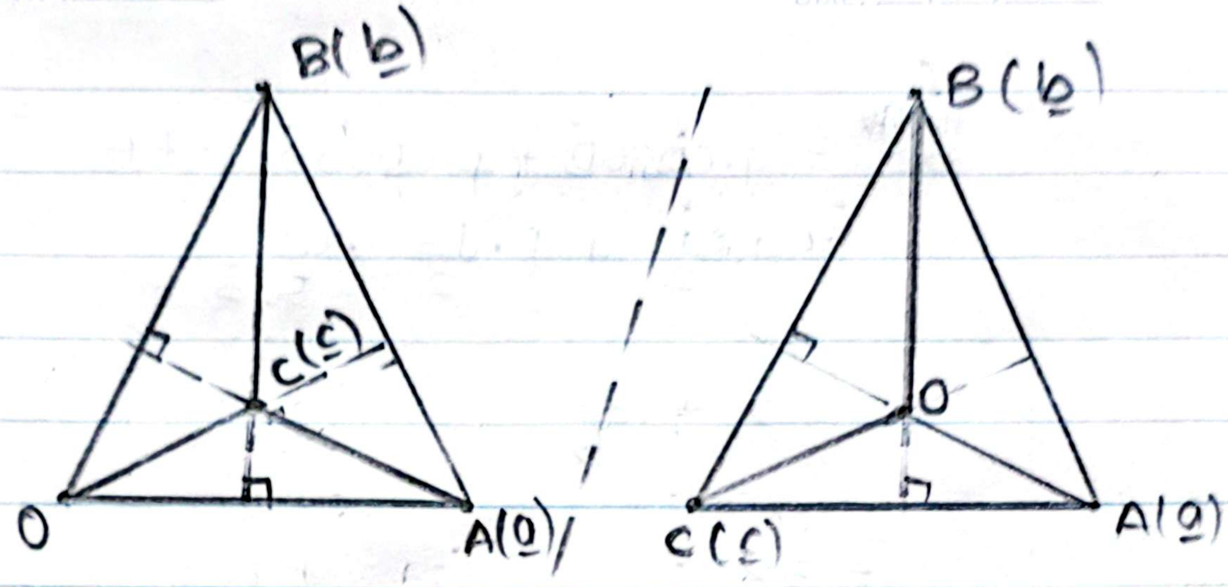
$\mu = 2\lambda$ $\qquad$ $\lambda = 4 - \lambda$

$$2\lambda = 4$$

$$\lambda = 2$$

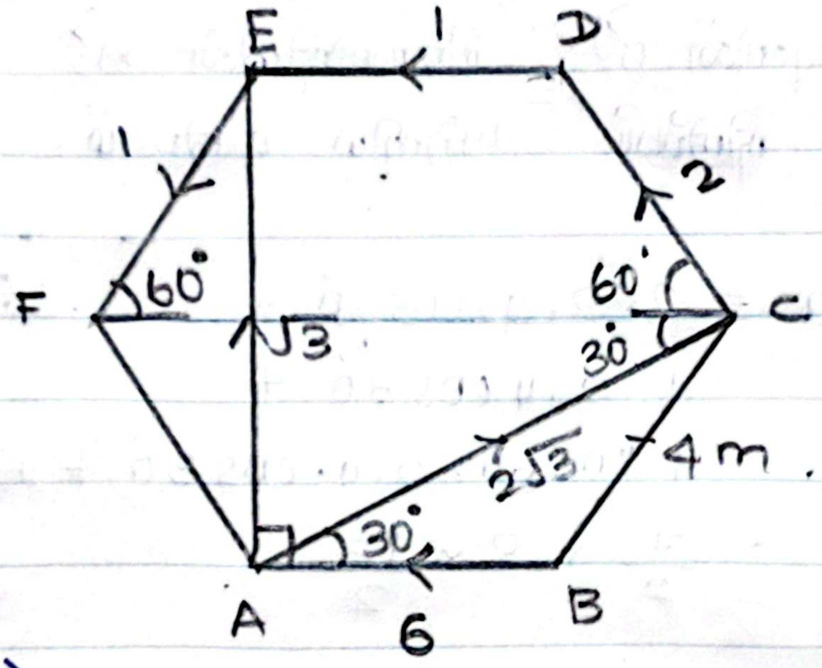
$$\mu = 4$$

14). (b).



$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= (\underline{b} - \underline{a})$$
$$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$$
$$= (\underline{c} - \underline{b}).$$
$$\overrightarrow{OA} \cdot \overrightarrow{BC} = \underline{a}(\underline{c} - \underline{b})$$
$$0 = \underline{a}\,\underline{c} - \underline{a}\,\underline{b} \quad \text{——} \quad ①.$$
$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$$
$$= \underline{c} - \underline{a}.$$
$$\overrightarrow{OB} \cdot \overrightarrow{AC} = \underline{b}(\underline{c} - \underline{a})$$
$$= \underline{b} \cdot \underline{c} - \underline{a} \cdot \underline{b} \quad \text{——} \quad ②.$$
$$①, ② \Rightarrow \underline{a} \cdot \underline{c} - \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{c} - \underline{a} \cdot \underline{b}.$$
$$\underline{a} \cdot \underline{c} - \underline{b} \cdot \underline{c} = 0$$
$$\underline{c}\,(\underline{a} - \underline{b}) = 0$$
$$\underline{c} \cdot (\underline{b} - \underline{a}) = 0$$
$$\overrightarrow{OC} \cdot \overrightarrow{AB} = 0$$
$$\therefore OC \perp \overrightarrow{AB}.$$

(c)



i) $\overrightarrow{AB} \Rightarrow 2\sqrt{3}\cos 30° - 6 - 2\cos 60 - 1 - 1\cos 60°$

$= 3 - 6 - 1 - 1 - \frac{1}{2}$

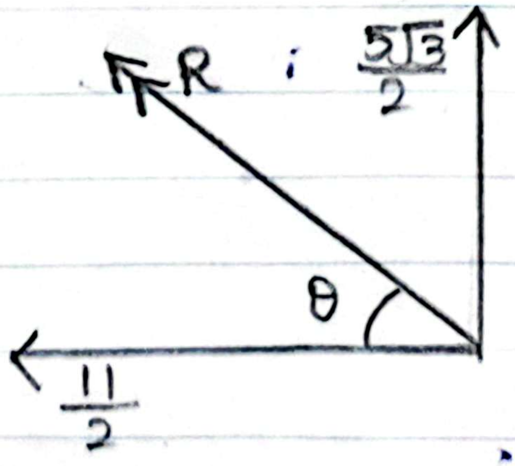
$= -\frac{11}{2}$

ii) $\overrightarrow{AE} \Rightarrow \sqrt{3} + 2\sqrt{3}\cos 60° + 2\cos 30° \cancel{\#} - 1\cos 30°$

$= 2\sqrt{3} + \sqrt{3} - \frac{\sqrt{3}}{2}$

$= \frac{5\sqrt{3}}{2}$

iii) $R^2 = \left(\frac{11}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2$

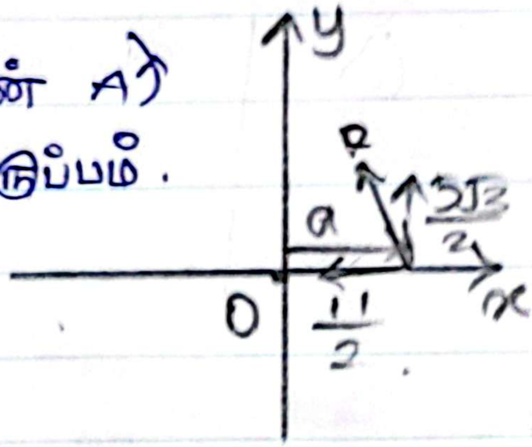


$= \frac{196}{4} = \left(\frac{14}{2}\right)^2$

$R = \frac{14}{2}$; $\tan\theta = \left(\frac{5\sqrt{3}}{11}\right)$

$R = 7$ $\therefore \theta = \tan^{-1}\left(\frac{5\sqrt{3}}{11}\right)$

iv) விளையுளின் O பற்றிய திருப்பம் = தொகுதியின் A பற்றிய திருப்பம்.



$$\frac{7\sqrt{3}}{2}\cdot a = 2\cdot 2\cdot 4\cdot\cos 30^\circ + 1\cdot 2\cdot 4\cos 30^\circ + 1\cos 60^\circ\cdot 2\cdot 4\cdot\cos 30^\circ \cancel{1\times 1\sin 60}$$

$$= \frac{7}{2}\times 8\times\frac{\sqrt{3}}{2}.$$

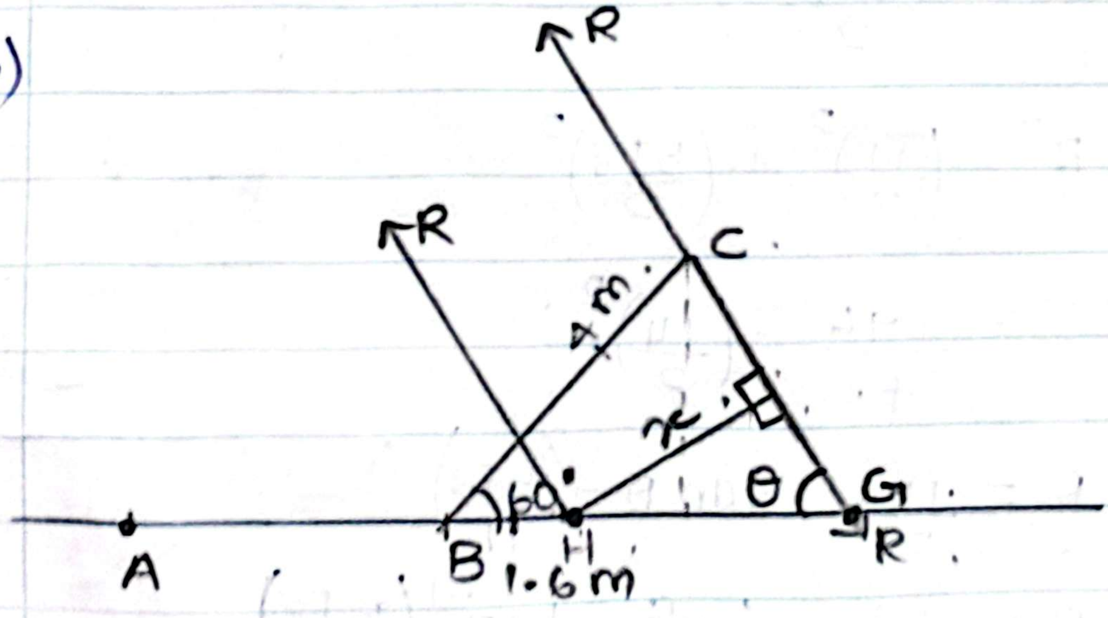
$$= 14\sqrt{3}.$$

$$a = \frac{14\times 2}{5}$$

$$= 5.6\ m.$$

நீட்டிய AB யை A யிலிருந்து 5.6m தூரத்தில் வெட்டும்.

v)



$BG = 4\cdot\cos 60° + 4\cos 30°\cdot\cot\theta$

$= 4\cdot\frac{1}{2} + 4\cdot\frac{\sqrt{3}}{2}\cdot\frac{11}{5\sqrt{3}}$

$= 2 + \frac{22}{5}$

$= \frac{32}{5}$

$= 6.4\,m$.

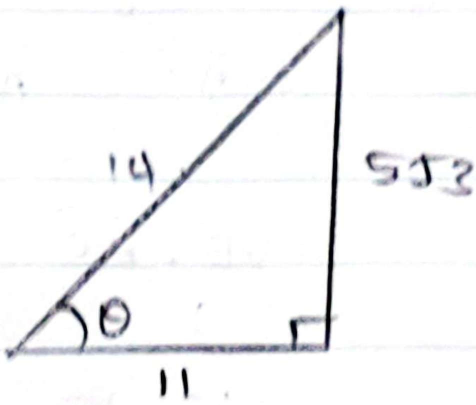
$HG = 6.4 - 1.6$

$= 4.8$.

$\sin\theta = \frac{x}{4.8}$

$x = 4.8 \times \frac{5\sqrt{3}}{14}$

$x = \frac{12\sqrt{3}}{7}$

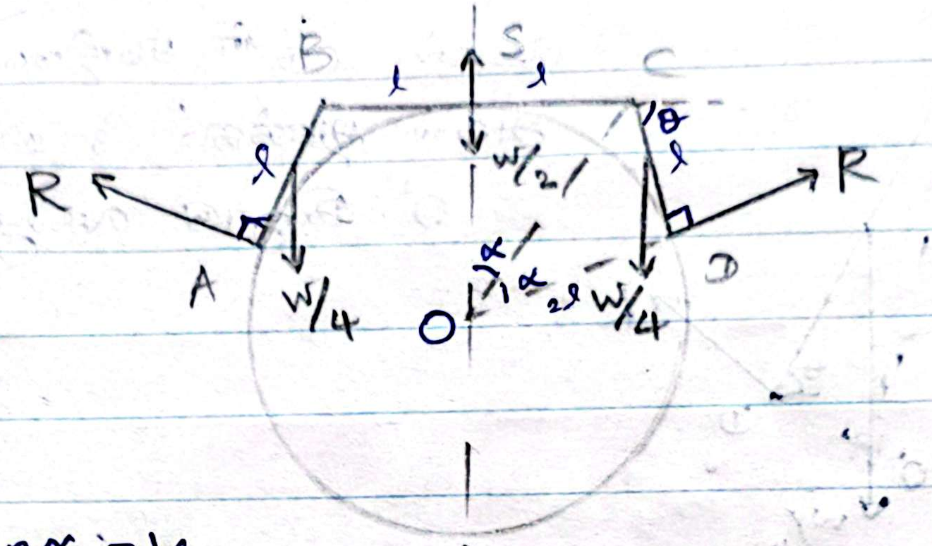


இணையின் பருமன் $= \frac{12\sqrt{3}}{7} \times \frac{14}{2}$

$= 12\sqrt{3}\,Nm$.

திருப்பம். $G = 12\sqrt{3}\,Nm$.

15)
a)



i) $\tan\alpha = \frac{1}{2}$

$\theta = 2\alpha$ ↺ සමමිතිකතාව

$\tan\theta = \tan 2\alpha$

$= \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \frac{4}{3}$

$(\overline{CD})$ C ↻ $Rl - \frac{W}{4}\left(\frac{l}{2}\cos\theta\right) = 0$

$R = \frac{W}{8}\cos\theta$

$= \frac{W}{8}\left(\frac{3}{5}\right)$

$R = \frac{3W}{40}$ ——— (1)

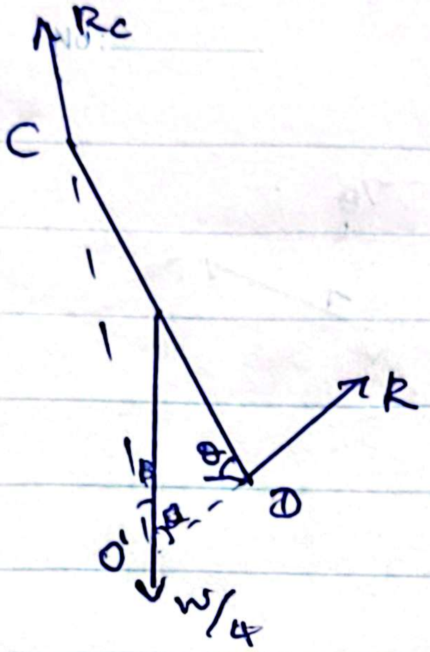
$(\overline{AB, BC, CD})$ ↓ $W - S - 2R\cos\theta = 0$

$S = W - 2\left(\frac{3W}{40}\right)\left(\frac{3}{5}\right)$

$S = W - \frac{9W}{100}$

$S = \frac{91W}{100}$

2)



கோல் CD ன் சமநிலைக்கு அதில் தாக்கும் 3 விசைகளும் O' இனூடு செல்லும்.

$\rightarrow$ $R\sin\theta - R_c\sin\beta = 0$

$\uparrow$ $R\cos\theta + R_c\cos\beta - W/4 = 0$

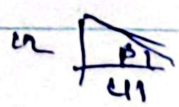
$R_c\cos\beta = W/4 - R\cos\theta$ ——— ①

$R_c\sin\beta = R\sin\theta$ ——— ②

②/① $\Rightarrow$ $\tan\beta = \dfrac{R\sin\theta}{W/4 - R\cos\theta}$

$$= \frac{\left(\frac{3W}{40}\right)\left(\frac{4}{5}\right)}{\frac{W}{4} - \left(\frac{3W}{40}\right)\left(\frac{3}{5}\right)}$$

$\tan\beta = 12/41$

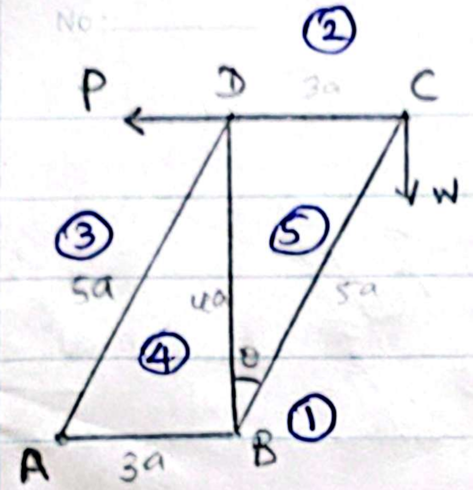


$\beta = \tan^{-1}(12/41)$

② $\Rightarrow$ $R_c = \dfrac{R\sin\theta}{\sin\beta} = \dfrac{\left(\frac{3W}{40}\right)\left(\frac{4}{5}\right)\sqrt{41^2+12^2}}{12}$

$R_c = \dfrac{\sqrt{73}}{40}$

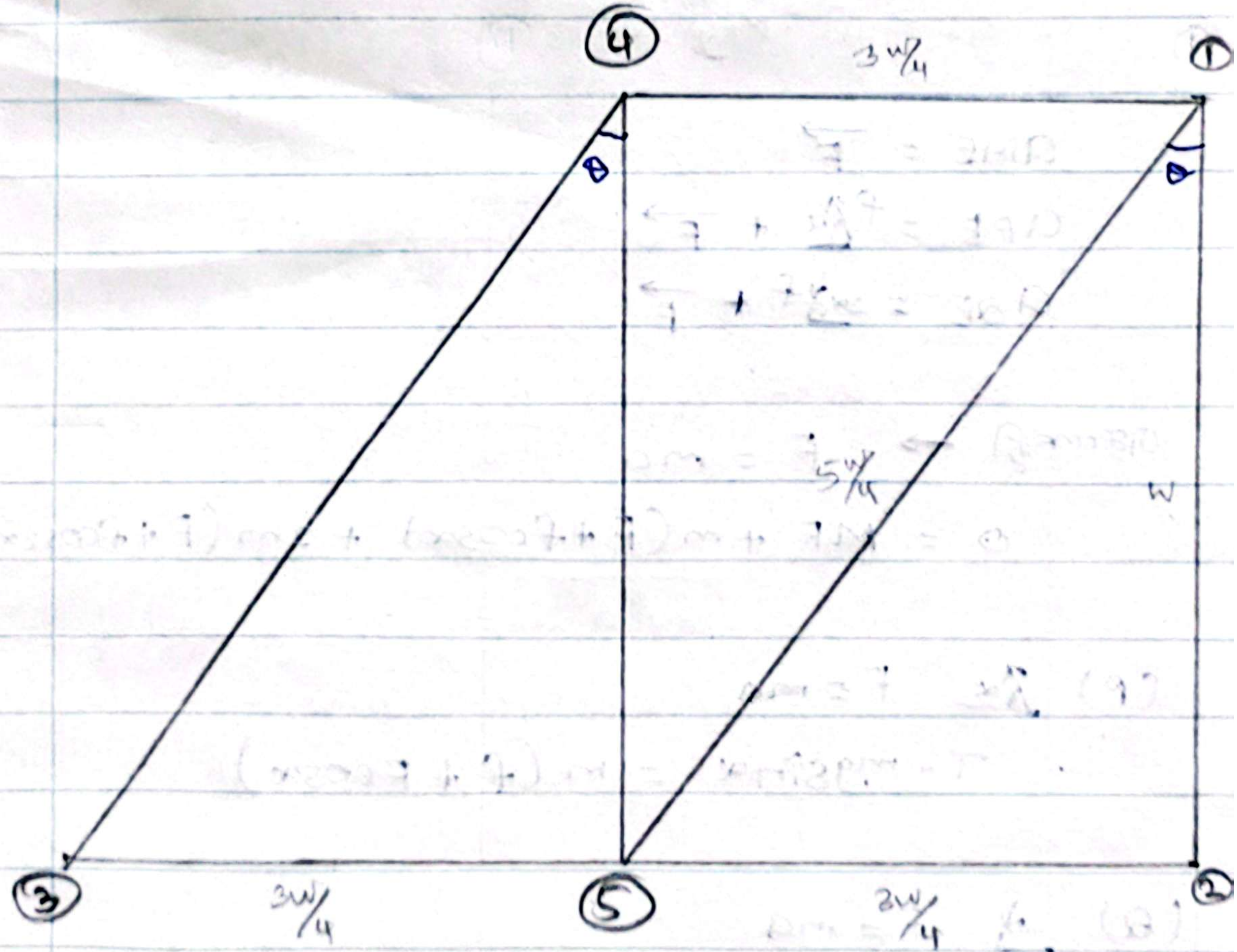
b)



தொகுதியின் சமநிலைக்கு

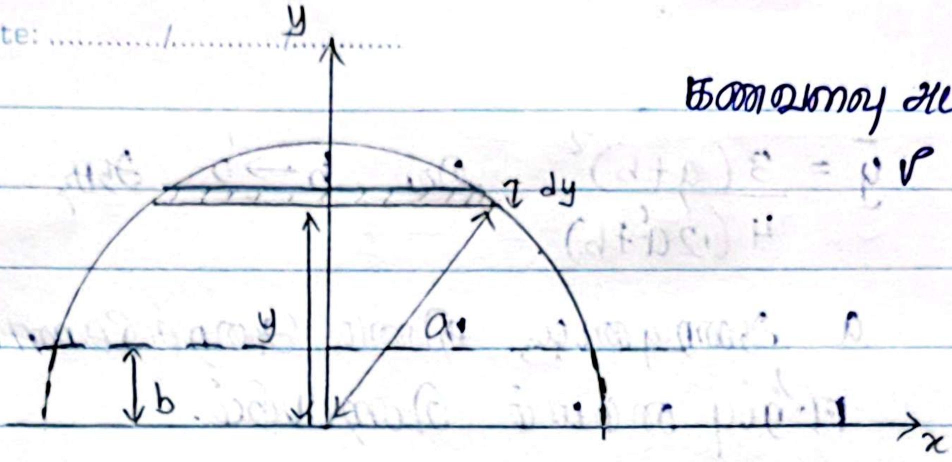
$A \circlearrowleft \quad W(6a) - P(4a) = 0$

$P = \frac{3W}{2}$



கோல்	இழுவை	உதைப்பு
AB	–	$\frac{3W}{4}$ N
BC	–	$\frac{5W}{4}$ N
CD	$\frac{3W}{4}$ N	–
BD	W N	–
AD	–	$\frac{5W}{4}$ N

(16)



கனவளவு அடர்த்தி ρ எனக

$$m_r = \pi\rho(a^2-y^2)\,dy \qquad , \quad y_r = y$$

$$\bar{y} = \frac{\sum_b^a \pi\rho(a^2-y^2)y\,dy}{\sum_b^a \pi\rho(a^2-y^2)\,dy}$$

$$= \frac{\int_b^a (a^2y - y^3)\,dy}{\int_b^a (a^2-y^2)\,dy}$$

$$= \frac{\frac{a^2}{2}\left[y^2\right]_b^a - \frac{1}{4}\left[y^4\right]_b^a}{a^2\left[y\right]_b^a - \frac{1}{3}\left[y^3\right]_b^a}$$

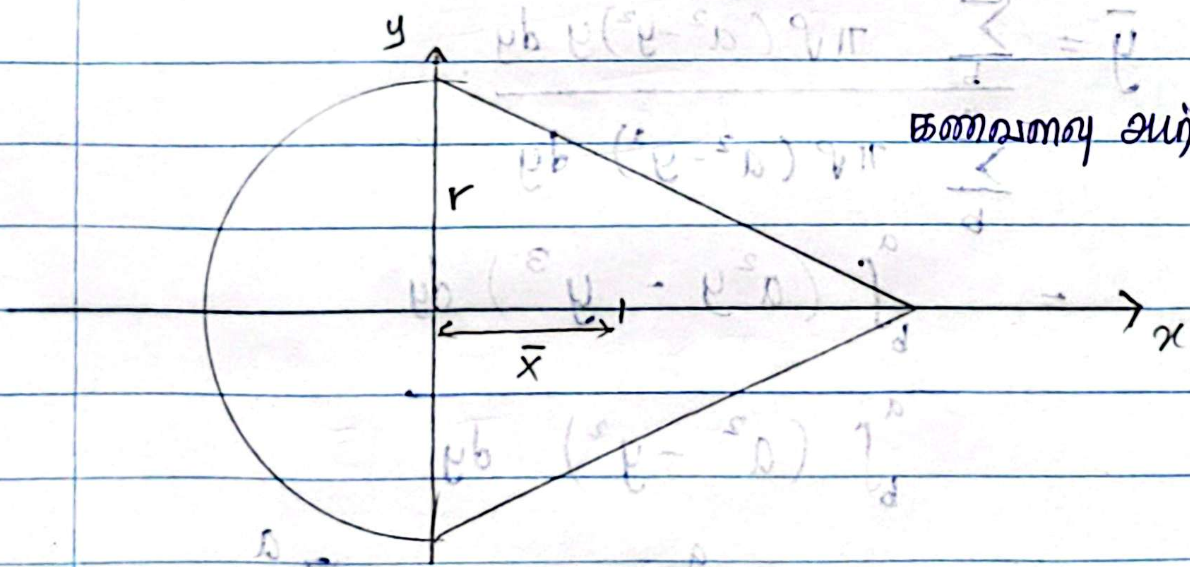
$$= \frac{\frac{a^2}{2}(a^2-b^2) - \frac{1}{4}(a^4-b^4)}{a^2(a-b) - \frac{1}{3}(a^3-b^3)}$$

$$\bar{y} = \frac{3}{4}\frac{(a+b)^2}{(2a+b)}$$

$\bar{y} = \dfrac{3(a+b)^2}{4(2a+b)}$ இல் $b \to 0$ ஆக

a ஆரையுடைய திண்ம அரைக்கோளத்தின் ஈர்ப்பு மையம் பெறப்படும்.

$$\bar{y} = \frac{3}{4}\left(\frac{a^2}{2a}\right) = \frac{3a}{8}$$



உருவம்	திணிவு	y அச்சிலிருந்து ஈர்ப்புமையதூரம்
கூம்பு	$\dfrac{\pi r^3 \tan\alpha \, \rho}{3}$	$\dfrac{r\tan\alpha}{4}$
அரைக்கோளம்	$\dfrac{2}{3}\pi r^3 \rho$	$\dfrac{3r}{8}$
சேர்த்தி உடல்	$\dfrac{\pi r^3 \rho\,(2+\tan\alpha)}{3}$	$\bar{x}$

y அச்சு பற்றி திருப்பம் , ②

$$\frac{\pi r^3 \rho}{3}(2+\tan\alpha)\,\bar{x} = \frac{\pi r^3 \rho \tan\alpha\,(r\tan\alpha)}{3\times 4} - \frac{2\pi r^3 \rho\,(3r)}{3\times 8}$$

$$(2+\tan\alpha)\,\bar{x} = \frac{r}{4}(\tan^2\alpha - 3)$$

$$\bar{x} = \frac{r\,|\tan^2\alpha - 3|}{8 + 4\tan\alpha}$$

a) $\tan^2\alpha < 3$, $(\tan^2\alpha - 3) < 0$

ஈர்ப்பு மையம் மறை x - அச்சில் காணப்படும்

$\therefore$ சமநிலை குழம்பும்

b) $\tan^2\alpha > 3$, $(\tan^2\alpha - 3) > 0$

ஈர்ப்பு மையம் நேர் x- அச்சில் காணப்படும்

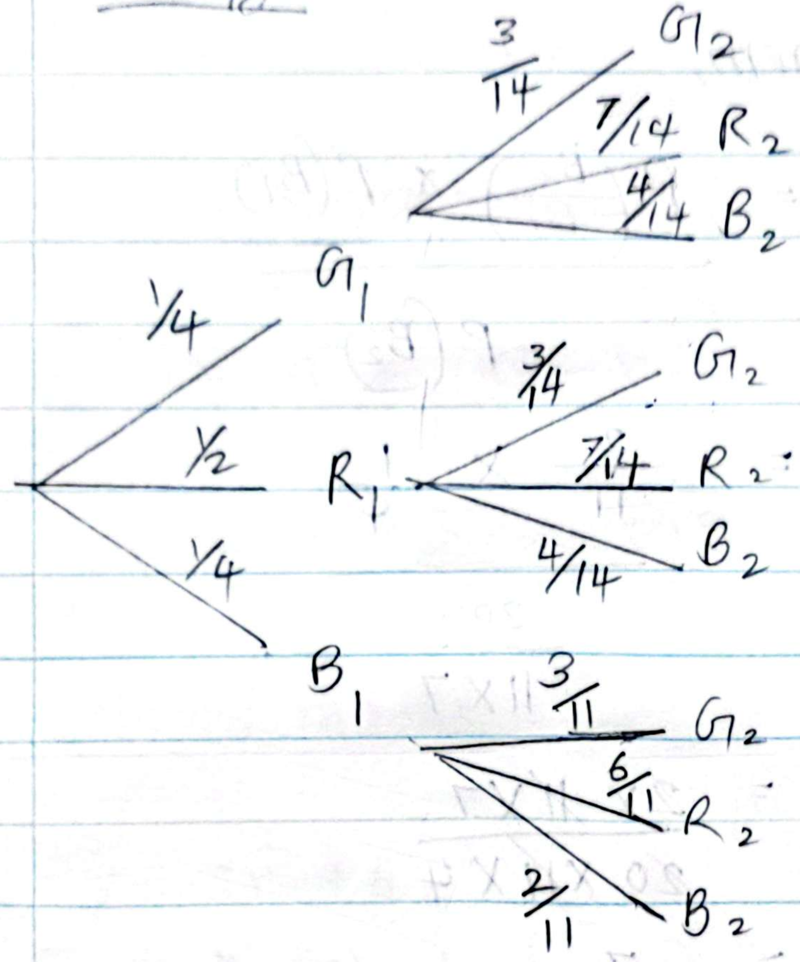
$\therefore$ சமநிலை குழம்பும்.

c) $\tan^2\alpha = 3$, $\bar{x} = 0$

$\therefore$ தொடர்ந்து சமநிலையில் காணப்படும்.

(17)

1ம் பந்து   2ம் பந்து



$$P(B_1) = \frac{1}{4}$$

$$P(B_2) = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{14} + \frac{1}{\cancel{2}} \times \frac{\cancel{4}^{2}}{14} + \frac{1}{\cancel{4}_{2}} \times \frac{\cancel{2}}{11}$$

$$= \frac{3}{2 \times 7} + \frac{1}{2 \times 11}$$

$$= \frac{20}{77}$$

Bay's Theorem,

$$P\left(\frac{B_1}{B_2}\right) = \frac{P\left(\frac{B_2}{B_1}\right) \times P(B_1)}{P(B_2)}$$

$$= \frac{\frac{2}{11} \times \frac{1}{4}}{\frac{20}{11 \times 7}}$$

$$= \frac{\cancel{2} \times \cancel{11} \times 7}{\cancel{20} \times \cancel{11} \times 4}$$

$$= \frac{7}{40}$$

b) $y_i = ax_i + b.$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{\sum_{i=1}^{n} (ax_i + b)}{n} = \frac{a\sum_{i=1}^{n} x_i}{n} + \frac{nb}{n}$$

$$\bar{y} = a\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) + b$$

$$\bar{y} = a\bar{x} + b$$

$$nS_y^2 = \sum_{i=1}^{n}(y_i - \bar{y})^2$$

$$= \sum_{i=1}^{n}(ax_i + b - a\bar{x} - b)^2$$

$$= a^2 \sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$S_y = |a| \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}} = |a| S_x$$

combined maths

$$|a| = \frac{S_y}{S_x} = \frac{20}{15}$$

$$\bar{y} = a\bar{x} + b$$

$$50 = \frac{4}{3} \times (m) + b \quad (1)$$

$$\boxed{a = \frac{4}{3}}$$

$$y = ax + b$$

$$40 = \frac{4}{3} \times 40 + b.$$

$$\boxed{b = \frac{-40}{3}}$$

$$(1) \Rightarrow \quad 50 = \frac{4}{3} \times m - \frac{40}{3}$$

$$\boxed{m = \frac{95}{2}}$$

CamScanner

Physics

$\bar{y} = a_1 \bar{x} + b_1$

$50 = a_1(45) + b_1$ — ②

$y = a_1 x + b_1$

$65 = a_1(61) + b_1$ — ③

③ − ② $\Rightarrow$ $15 = 16\, a_1$

$a_1 = \frac{15}{16}$

$\frac{S_y}{S_x} = |a_1| = \frac{20}{p} = \frac{15}{16}$

$$\boxed{p = \frac{64}{3}}$$

ஆரம்பத்தில் 1% ஆனோரின் மொத்த புள்ளிகள் $= T_1$ எனக.

$\frac{T_1}{(n/100)} = 60$

$$\boxed{T_1 = \frac{60n}{100}}$$

எஞ்சிய 99% ஆனோரின் மொத்த புள்ளிகள் $= T_2$ என்க.

$$\frac{T_1 + T_2}{n} = 50$$

$$T_2 = 50n - \frac{60n}{100}$$

மீளாய்வின் பின்னர் 1% ஆனோரின் மொத்த புள்ளிகள் $= T_3$ என்க.

$$\frac{T_3}{n/100} = 64 \Rightarrow \boxed{T_3 = \frac{64n}{100}}$$

மீளாய்வின் பின்னர் மொத்த C.M மாணவர்களின் புள்ளிகளின் இடை $\Rightarrow$

$$இடை = \frac{T_3 + T_2}{n}$$

$$= \frac{\frac{64n}{100} + \frac{5000n}{100} - \frac{60n}{100}}{n}$$

$$= 50.04$$