பகுதி A

01) $n=1$ ஆக $\Rightarrow f(1) = 7^3+8^3$

$= 855$ (5)

$= 15 \times 57$

$\therefore n=1$ ற்கு முடிவு உண்மையாகும்.

$n=p$ ற்கு முடிவு உண்மை என்க. [இங்கு $p \in \mathbb{Z}^+$]

$7^{(p+2)} + 8^{(2p+1)} = 57K$ (5) [இங்கு $K \in \mathbb{Z}^+$]

$n=1$ ஆக $\Rightarrow$

$$7^{(p+1)+2} + 8^{2(p+1)+1} = 7^{(p+3)} + 8^{(2p+3)} \quad (5)$$
$$= 7\cdot 7^{(p+2)} + 64\cdot 8^{(2p+1)}$$
$$= 7\left[7^{(p+2)} + 8^{(2p+1)}\right] + 57\cdot 8^{(2p+1)}$$
$$= 57\left[7K + 8^{(2p+1)}\right]$$

$\therefore n=p+1$ ற்கு முடிவு உண்மையாகும் (5)

$\therefore$ கணிதத் தொகுத்தறிவு முறைப்படி எல்லா $n \in \mathbb{Z}^+$ ற்கும் முடிவு உண்மையாகும். (5)

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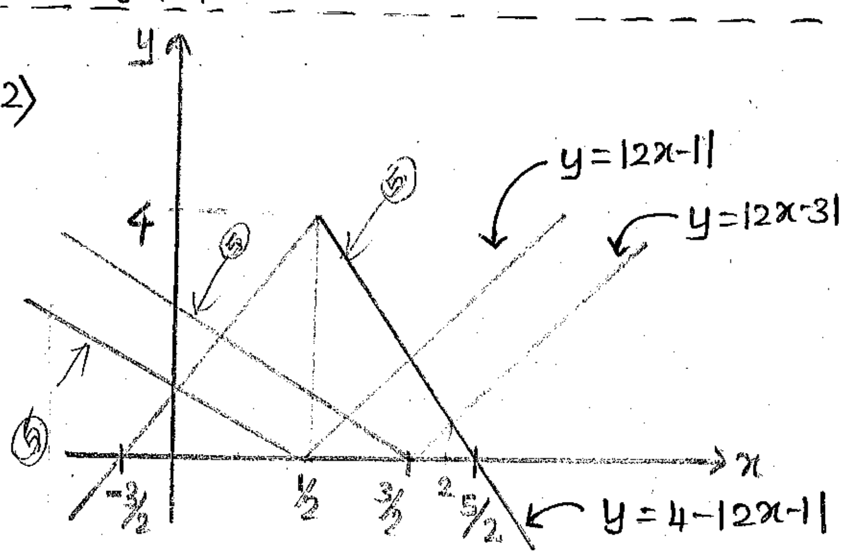
02)



$y=|2x-1|$

$x>\frac{1}{2} \Rightarrow y=2x-1$

$x<\frac{1}{2} \Rightarrow y=1-2x$

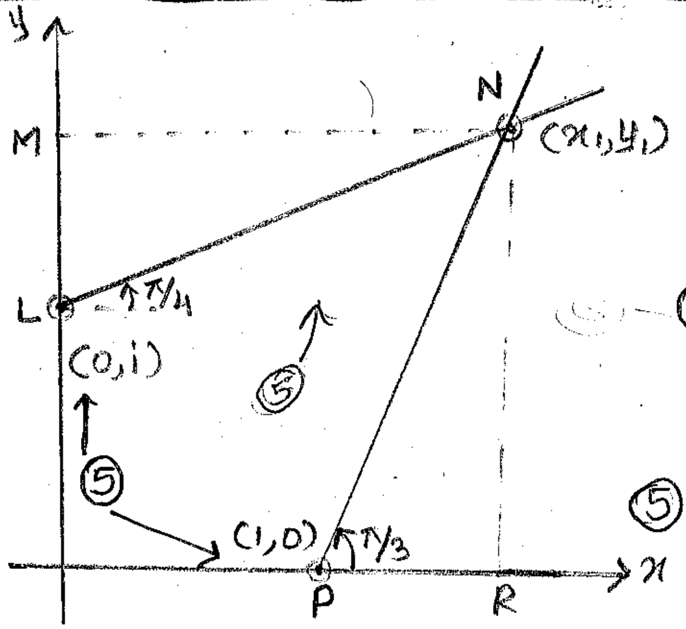
$|2x-1| < 4-|2x-3|$

$\Leftrightarrow |2x-3| < 4-|2x-1|$

$\Leftrightarrow 0<x<2$ (5)

(5)

03)



Δ PRN இல் Sin விதிப்படி ;

$$\frac{y_1}{\sin \pi/3} = \frac{(x_1 - 1)}{\sin \pi/6}$$

5 — $y_1 = \sqrt{3}(x_1 - 1)$ — ①

Δ MLN இல் Sin விதிப்படி ;

5 — $\dfrac{(y_1 - 1)}{\sin \pi/4} = \dfrac{x_1}{\sin \pi/4}$

$y_1 = (x_1 + 1)$ — ②

①, ② $\Rightarrow$ ① − ② $\Rightarrow x_1(\sqrt{3} - 1) = 1 + \sqrt{3}$

$$x_1 = \frac{1+\sqrt{3}}{\sqrt{3}-1}$$

$x_1 = K$ — 5

$\therefore\ y_1 = (K+1)$

$$|z - (x_1 + iy_1)| = x_1$$

$$|z - [K + i(K+1)]| = K$$

$$|z - K - (K+1)i| = K$$

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04) $T_{r+1} = {}^6C_r\, (Kx)^{6-r} \left(\tfrac{1}{x}\right)^r$ — 5

$$= {}^6C_r\, K^{(6-r)}\, x^{(6-2r)}$$

$r = 3$ ஆக மாறிலி உறுப்பு பெறப்படும் — 5

$r = 3 \Rightarrow T_4 = {}^6C_3\, K^3$

$$= 540$$

$\dfrac{6!}{3! \cdot 3!} K^3 = 540$ — 5

$K^3 = \dfrac{540}{20}$ — 5

$$K^3 = 27$$

$$K^3 = 3^3$$

$K = 3$ — 5

05) $L.H.S = \lim_{\theta \to \pi/4} \frac{\sqrt{\tan\theta} - 1}{(2\sqrt{\theta} - \sqrt{\pi})}$

$= \lim_{\theta \to \pi/4} \frac{\tan\theta - 1}{4\theta - \pi} \times \left[\frac{\sqrt{4\theta} + \sqrt{\pi}}{\sqrt{\tan\theta} + 1}\right]$ — 10

$= \frac{\sqrt{2}}{4} \lim_{\theta \to \pi/4} \frac{\sin\theta\cos\pi/4 - \cos\theta\sin\pi/4}{(\theta - \pi/4)} \frac{\times(\sqrt{4\theta} + \sqrt{\pi})}{\cos\theta(\sqrt{\tan\theta} + 1)}$

5

$= \frac{\sqrt{2}}{4} \lim_{\theta \to \pi/4} \frac{\sin(\theta - \pi/4)}{(\theta - \pi/4)} \times \lim_{\theta \to \pi/4} \frac{\sqrt{4\theta} + \sqrt{\pi}}{(\sqrt{\tan\theta} + 1)\cos\theta}$ — 5

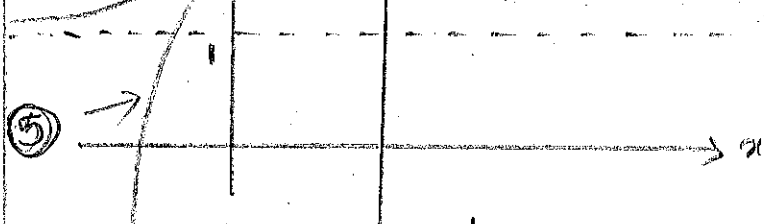
$= \frac{\sqrt{2}}{4} \times 1 \times \frac{2\sqrt{\pi}}{2} \times \frac{1}{\sqrt{2}}$ — 5

5

$= \frac{\sqrt{\pi}}{2}$

$\boxed{25}$

06)



$S = \int_0^1 (e^x + 1)\,dx - \int_0^1 [2 + \ln(x+1)]\,dx$

5

$= \int_0^1 (e^x - 1)\,dx - \int_0^1 \ln(x+1)\,dx$

$= [e^x - x]_0^1 - [2\ln 2 - 1]$

$= (e - 1) - (1 - 0) - (2\ln 2 - 1)$ — 5

2

$= e - (2\ln 2 + 1)$

$\int_0^1 \ln(x+1)\,dx = \int_0^1 \ln(x+1) \frac{d(x+1)}{dx}\,dx$

$= \left[(x+1)\ln(x+1)\right]_0^1 - \int_0^1 (x+1)\frac{1}{(x+1)}\,dx$

$= 2\ln 2 - \int_0^1 (x+1)\frac{1}{(x+1)}\,dx$

$= 2\ln 2 - \int_0^1 (1)\,dx$

$= 2\ln 2 - [x]_0^1$

$= 2\ln 2 - 1$

5

$\boxed{25}$

07) $x = t - \sin t$ $\qquad\qquad$ $y = 1 - \cos t$

$\dfrac{dx}{dt} = 1 - \cos t$ $\qquad\qquad$ $\dfrac{dy}{dt} = \sin t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \sin t \cdot \frac{1}{(1-\cos t)} \quad \text{— } 5$$

$$\left(\frac{dy}{dx}\right)_{t=\pi/4} = \frac{1/\sqrt{2}}{1 - 1/\sqrt{2}}$$

$$= \frac{1}{(\sqrt{2}-1)} \quad \text{— } 5$$

$t = \pi/4 \Leftrightarrow x = \left(\pi/4 - 1/\sqrt{2}\right)$ — 5

$\Leftrightarrow y = 1 - 1/\sqrt{2}$

$t = \pi/4$ இல் தொடலி $\Rightarrow \dfrac{y - (1 - 1/\sqrt{2})}{x - (\pi/4 - 1/\sqrt{2})} = \dfrac{1}{(\sqrt{2}-1)}$ — 5

$$4(\sqrt{2}-1)y - \frac{4(\sqrt{2}-1)^2}{\sqrt{2}} = 4x - \pi + 2\sqrt{2}$$

$$4(\sqrt{2}-1)y - 4x - 2(3\sqrt{2}-4) - 2\sqrt{2} + \pi = 0$$

$$4(\sqrt{2}-1)y - 4x - 8(\sqrt{2}-1) + \pi = 0 \quad \text{— } 5$$

$\boxed{25}$

08) $P \equiv \left[\dfrac{2\times 10\cos\theta + 3\times 5}{5}, \dfrac{2\times 10\sin\theta + 3\times 0}{5}\right]$

$\equiv \left[(4\cos\theta + 3), 4\sin\theta\right]$

5 5

$x = 4\cos\theta + 3$

$(x-3) = 4\cos\theta$ —— ①

$y = 4\sin\theta$ —— ②

$①^2 + ②^2 \Rightarrow x^2+y^2-6x+9 = 16\left[\sin^2\theta+\cos^2\theta\right]$

$x^2+y^2-6x-7=0$ —— 5

∴ P இன் ஒழுக்கு வட்டமாகும்.

மையம் $\equiv (+3, 0)$ —— 5

ஆரை $= \sqrt{9+0-(-7)}$

$= \sqrt{16}$ —— 5

$= 4$

$\boxed{25}$

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09) வட்டம் இவ்வாறு அமையும்

$S \equiv x^2+y^2+2gx+2fy+c=0$

$2g(-3a)+2f(0) = c+5a^2$ —— ① —— 5

$-6ag = c+5a^2$

$2g(0)+2f(-3a) = c+5a^2$ —— 5

$-6af = c+5a^2$ —— ②

$g = f$ —— 5

$c = -6ag-5a^2$ —— 5

$S \equiv x^2+y^2+2gx+2gy+(-6ag-5a^2)=0$

$S \equiv x^2+y^2-5a^2+2g(x+y-3a) = 0$ 5

$S \equiv x^2+y^2-5a^2+2k(x+y-3a)=0$ [இங்கு $k=g$]

$\boxed{25}$

10) (i) $2S \cdot \sin\theta = 2\sin\theta\left[\sin\theta + \sin3\theta + \sin5\theta + \sin7\theta\right]$

$$= 2\sin\theta\left[2\sin4\theta\cos3\theta + 2\sin4\theta\cos\theta\right]$$

$$= 4\sin\theta\sin4\theta\left[\cos3\theta + \cos\theta\right]$$

$$= 4\sin\theta\sin4\theta \times 2\cos2\theta \times \cos\theta \quad \text{—— } 5$$

$$= 4\sin4\theta\cos2\theta \times 2\cos\theta\sin\theta$$

$$= 4\sin4\theta\cos2\theta \times \sin2\theta$$

$$= 2\sin^2 4\theta \quad \text{—— } 5$$

$$= 1 - \cos8\theta$$

(ii) $L = 2\cos4\theta\cos3\theta + 2\cos4\theta\cos\theta$

$$\left.\begin{aligned} &= 2\cos4\theta\left[\cos3\theta + \cos\theta\right] \\ &= 2\cos4\theta \times 2\cos2\theta\cos\theta \\ &= 4\cos\theta\cos2\theta\cos4\theta \end{aligned}\right\} 5$$

$$\frac{S}{L} = \frac{\dfrac{(1-\cos8\theta)}{2\sin\theta}}{4\cos\theta\cos2\theta\cos4\theta} \quad \text{—— } 5$$

$$= \frac{2\sin^2 4\theta}{4 \times 2\sin\theta\cos\theta\cos2\theta\cos4\theta}$$

$$= \frac{\sin^2 4\theta}{4 \times \sin2\theta\cos2\theta\cos4\theta}$$

$$\left.= \frac{\sin^2 4\theta}{\sin4\theta\cos4\theta}\right\} 5$$

$$= \tan4\theta$$

11) a) $L(x) = x^3 + ax^2 + bx - 12$

(i)

$L(-2) = -150$ —— 5

$-8 + 4a - 2b - 12 = -150$

$4a - 2b = -130$

$2a - b = -65$

$2a - b + 65 = 0$ —— ① —— 5

(ii)

$L(3) = 0$ —— 5

$27 + 9a + 3b - 12 = 0$

$9a + 3b + 15 = 0$

$3a + b + 5 = 0$ —— ② —— 5

① + ② $\Rightarrow$ $5a = -70$

$a = -14$ —— 5

$a = -14$ ඔබ

① $\Rightarrow$ $2 \times -14 - b + 65 = 0$

$b = 37$ —— 5

$\therefore L(x) = x^3 - 14x^2 + 37x - 12$

$S(2) = 0$ —— 5

(*) $p\,[2^3 - 14(2^2) + 37(2) - 12] + 28q = 0$

$p + 2q = 0$ —— 5

$(x = -1)$ ஆக $\quad S(-1) = -156$ —— 5

$$p(-1-14-37-12) + 28q = -156$$

$$-16p + 7q = -39 \quad \text{—— } 5$$

$$32q + 7q = -39 \quad (\because 3)$$

$$q = (-1) \quad \text{—— } 5$$

$\therefore$ 3 $\Rightarrow p = -2q$

$\quad = 2$ —— 5

$$S(x) = p \cdot L(x) + 28q$$

$\therefore$ மீதி $= 28q$

$\quad = (-28)$ —— 5

(b) $x^2 - 2(a-2)x + 2a - 10 = 0$ —— ①

$\Delta = 4(a-2)^2 - 4(1)(2a-10)$ —— 5

$= 4(a^2 - 4a + 4 - 2a + 10)$

$= 4(a^2 - 6a + 14)$

$= 4[(a-3)^2 + 5]$ —— 5

$(a-3)^2 \geq 0$ —— 5

$\therefore \Delta > 0$

ஆகவே மெய்யான மூலங்களைக் கொண்டிருக்கும் —— 5

① இன் மூலங்கள் α_1, β_1 என்க

$\alpha_1 + \beta_1 = 2(a-2)$ —— 5

$\alpha_1 \beta_1 = 2a - 10$ —— 5

$\alpha_1 = -\beta_1$ (தரவு) —— 5

$\alpha_1 + \beta_1 = 0$

$2(a-2) = 0$

$a = 2$ —— 5

$|\alpha_1 - \beta_1| = 6$ (தரவு) —— 5

$(\alpha_1 - \beta_1)^2 = 36$

$(\alpha_1 + \beta_1)^2 - 4\alpha_1\beta_1 = 36$ —— 5

$[2(a-2)]^2 - 4(2a-10) = 36$

$4(a^2 - 4a + 4) - 8a + 40 = 36$

$a^2 - 6a + 5 = 0$

$(a-5)(a-1) = 0$

$a = 5$ or $a = 1$ —— 5

$(a+2b)x^2+2(a-b)x+(a-4b)=0$

$\Delta = 4(a-b)^2-4(a+2b)(a-4b)$ — (5)

$= 4[a^2-2ab+b^2-a^2+2ab+8b^2]$

$= 36b^2$

$= (6b)^2$

$(6b)^2 \geq 0$

$\therefore \Delta \geq 0$ ஆகவே, இவ்விருபடிச்சமன்பாடு மெய்மூலங்களை கொண்டிருக்கும் — (5)

மூலங்கள் சமமானவை எனின் $\Delta=0$ ஆகும் (5)

$\therefore (6b)^2 = 0$

$b = 0$

$2x^2+x+5=0$ இன் மூலங்கள் α, β எனின்

$\alpha+\beta = -\frac{1}{2}$ — (3)

$\alpha\beta = \frac{5}{2}$ — (4)

(5)

$2x^2-3x+2k=0$ இன் மூலங்கள் $(\alpha+1), (\beta+1)$ எனின்

$(\alpha+1)(\beta+1) = k$ — (5)

$\alpha\beta+\alpha+\beta+1 = k$

$\frac{5}{2} - \frac{1}{2} + 1 = k$

$\therefore k = 3$ — (5)

12) (a) A-5, B-5, C-5

(i) $^5C_2 \cdot {^5C_2} \cdot {^5C_2} = 10 \times 10 \times 10$
$= 1000$ —— 15

(ii) $^4C_1 \cdot {^4C_1} \cdot {^4C_1} = 4 \times 4 \times 4$
$= 64$ —— 15

(iii) $^{15}C_6 - {^3C_2} \cdot {^{10}C_6} = 5005 - 630$
$= 4375$ —— 15

(iv) $^{15}C_6 - {^3C_1} \cdot {^5C_5} \cdot {^{10}C_1} = 5005 - 30$
$= 4975$ —— 15

(b) $8x^3 + 2x^2 + 1 = Ax^2(2x+1) + B(x+1)^2(2x-1)$

x^3 கெழு $\Rightarrow$ $8 = 2A + 2B$
$A + B = 4$ —— 5

x^0 கெழு $\Rightarrow$ $1 = -B$
$B = -1$ —— 5
$\therefore A = 5$ —— 5

$8x^3 + 2x^2 + 1 = 5x^2(2x+1) - (x+1)^2(2x-1)$

$$U_r = \frac{8r^3+4r^2+1}{5^{(r+1)}(2r+1)(2r-1)}$$

$$= \frac{5r^2(2r+1) - (r+1)^2(2r-1)}{5^{(r+1)}(2r+1)(2r-1)}$$

$$U_r = \frac{r^2}{5^r(2r-1)} - \frac{(r+1)^2}{5^{(r+1)}(2r+1)} \quad \text{—— } 10$$

$$U_r = f(r) - f(r+1)$$

இங்கு $f(r) = \dfrac{r^2}{5^r(2r-1)}$ —— 10

$$\left.\begin{aligned} r=1 &\Rightarrow U_1 = f_1 - f_2 \\ r=2 &\Rightarrow U_2 = f_2 - f_3 \end{aligned}\right\} \; 10$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\left.\begin{aligned} r=(n-1) &\Rightarrow U(n-1) = f(n-1) - f(n) \\ r=n &\Rightarrow U_n = f_n - f(n+1) \end{aligned}\right\} \; 10$$

$$\sum_{r=1}^{n} U_r = f_1 - f(n+1) \quad \text{—— } 10$$

$$= \frac{1}{5} - \frac{(n+1)^2}{5^{(n+1)}(2n+1)} \quad \text{—— } 10$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} U_r = \lim_{n\to\infty}\left[\frac{1}{5} - \frac{(n+1)^2}{5^{(n+1)}(2n+1)}\right] \quad \text{—— } 5$$

$$= \frac{1}{5} \quad \text{—— } 5$$

∴ முடிவில்தொடர் $\sum_{r=1}^{n} U_r$ ஒருங்கும். கூட்டுத்தொகை $1/5$.

—— 5

13) a) (i) $A \times B = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$ 5

$= \begin{pmatrix} 4 & 9 \\ 2 & 3 \end{pmatrix}$

$B \times A = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$ 5

$= \begin{pmatrix} 7 & 6 \\ 1 & 0 \end{pmatrix}$

(இங்கு $A + B \neq B \times A$) 5

$A^2 = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ 5

$= \begin{pmatrix} 3 & 5 \\ -5 & 8 \end{pmatrix}$

$f(A) = \begin{pmatrix} 3 & 5 \\ -5 & 8 \end{pmatrix} - 5\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 5

$= \begin{pmatrix} 10 & 5 \\ -5 & 15 \end{pmatrix} - \begin{pmatrix} 10 & 5 \\ 5 & 15 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ 5

$= 0$

$$A^2 - 5A + 7I = 0$$

$$5A - A^2 = 7I$$

$$A\left(\tfrac{1}{7}\right)(5I - A) = I \quad 5$$

$$A^{-1} = \frac{1}{7}(5I - A) \quad 10$$

$$= \frac{1}{7}\left[5\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}\right]$$

$$= \frac{1}{7}\left[\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix} - \begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}\right]$$

$$= \frac{1}{7}\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix} \quad 5$$

(b) (i) $\frac{1}{1-z} = \frac{1}{(1-(\cos\theta - i\sin\theta))} \times \frac{(1-\cos\theta)+i\sin\theta}{(1-\cos\theta+i\sin\theta}$ — 5

$= \frac{(1-\cos\theta)+i\sin\theta}{(1-\cos\theta)^2 - i^2\sin^2\theta}$

$= \frac{1-\cos\theta+i\sin\theta}{1-2\cos\theta+\cos^2\theta+\sin^2\theta}$ $(\because i^2=-1)$ — 5

$= \frac{1-\cos\theta+i\sin\theta}{2(1-\cos\theta)}$

$= \frac{1}{2} + \frac{1}{2} \times \frac{i\sin\theta}{(1-\cos\theta)} \times \frac{(1+\cos\theta)}{(1+\cos\theta)}$ 10

$= \frac{1}{2} + \frac{1}{2} \frac{[i\sin\theta(1+\cos\theta)]}{\sin^2\theta}$

$= \frac{1}{2}\left[1 + \frac{i\,2\cos^2\theta/2}{2\sin\theta/2\,\cos\theta/2}\right]$

$= \frac{1}{2}\left[1+i\cot\theta/2\right]$ — 5

$z = \cos\theta + i\sin\theta$ — 1

$\frac{1}{z} = \frac{1}{(\cos\theta+i\sin\theta)} \times \frac{(\cos\theta-i\sin\theta)}{(\cos\theta-i\sin\theta)}$ — 5

$= \frac{\cos\theta - i\sin\theta}{(\cos^2\theta+\sin^2\theta)}$ $(\because i^2=-1)$

$\frac{1}{z} = \cos\theta - i\sin\theta$ — 2 — 5

① + ② $\Rightarrow z + \frac{1}{z} = 2\cos\theta$ —— ⑤

$$\frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta} = \frac{(\cos\alpha + i\sin\alpha)}{(\cos\beta + i\sin\beta)} \times \frac{(\cos\beta - i\sin\beta)}{(\cos\beta - i\sin\beta)}$$ —— ⑤

$$= \frac{(\cos\alpha\cos\beta + \sin\alpha\sin\beta) + i(\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{\cos^2\beta + \sin^2\beta}$$

$(\because i^2 = -1)$ —— ⑤

$$= \cos(\alpha - \beta) + i\sin(\alpha - \beta)$$ —— ⑤

$$z_1 = -1 + i$$

$$= \sqrt{2}\left[-\frac{1}{\sqrt{2}} + i\left(\frac{1}{\sqrt{2}}\right)\right]$$ —— ⑤

$$= \sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$$

$$\left.\begin{aligned} |z_1| &= \sqrt{2} \\ \arg(z_1) &= \frac{3\pi}{4} \end{aligned}\right\}$$ ⑤

$$z_2 = 1 + \sqrt{3}\,i$$

$$= 2\left[\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$$ —— ⑤

$$= 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$$

$$|z_2| = 2$$

$$\arg(z_2) = \frac{\pi}{3}$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

⑤

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

$$= \left(\frac{3\pi}{4} - \frac{\pi}{3}\right)$$

$$\frac{z_1}{z_2} = \frac{1}{\sqrt{2}}\left[\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)\right] \text{ ——— } 5$$

$$Re\left(\frac{z_1}{z_2}\right) = \frac{1}{\sqrt{2}}\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)$$

$$= \frac{1}{\sqrt{2}}\left[\cos\frac{3\pi}{4}\cos\frac{\pi}{3} + \sin\frac{3\pi}{4}\sin\frac{\pi}{3}\right]$$

$$= \frac{1}{\sqrt{2}}\left[-\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right]$$

$$= \frac{\sqrt{3}-1}{4}$$

5

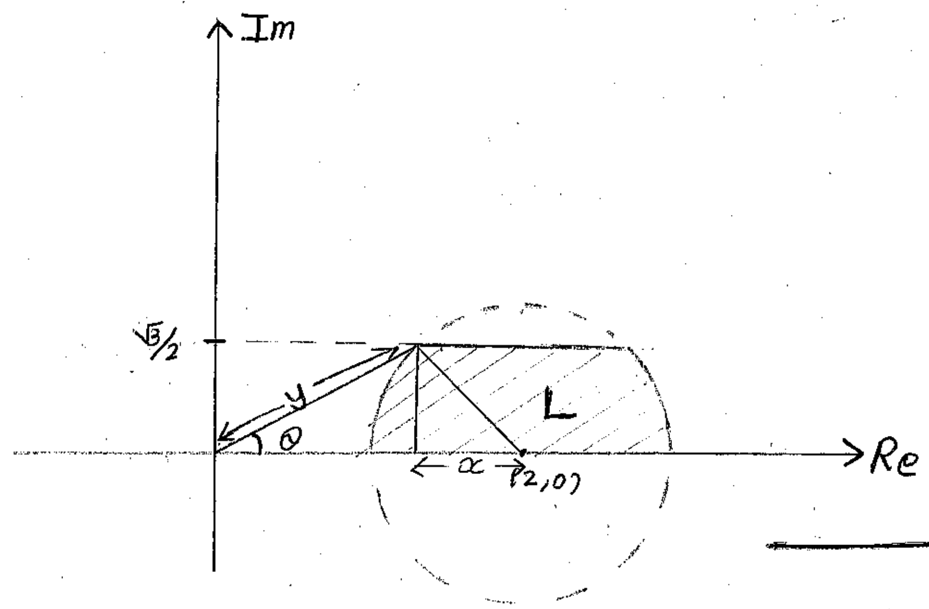
$$Im\left(\frac{z_1}{z_2}\right) = \frac{1}{\sqrt{2}}\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\sin\frac{3\pi}{4}\cos\frac{\pi}{3} - \cos\frac{3\pi}{4}\sin\frac{\pi}{3}\right)$$

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right]$$

$$= \frac{\sqrt{3}+1}{4}$$

5



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$x^2 = 1^2 - 3/4$

$x = 1/2 \quad (x > 0)$

$\tan\theta = \dfrac{\sqrt{3}/2}{3/2}$

$= 1/\sqrt{3}$

$\theta = \pi/6$

$\dfrac{\sqrt{3}/2}{y} = \sin \pi/6$

$y = \dfrac{\sqrt{3}/2}{1/2}$

$= \sqrt{3}$

$z = \sqrt{3}\left(\cos \pi/6 + i \sin \pi/6\right)$

$= \sqrt{3}\left[\dfrac{\sqrt{3}}{2} + i\left(\dfrac{1}{2}\right)\right]$

$= \dfrac{3}{2} + i\left(\dfrac{\sqrt{3}}{2}\right)$ 5

14) a) I $y = \frac{1}{2}(e^{x} + e^{-x})$

$$\frac{dy}{dx} = \frac{1}{2}\left[e^{x} - e^{-x}\right] \text{ ——— } 5$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(e^{x} - e^{-x})^2 \text{ ——— } ①$$

$$y^2 = \frac{1}{4}(e^{x} + e^{-x})^2 \text{ ——— } ②$$

$$① - ② \Rightarrow \left(\frac{dy}{dx}\right)^2 - y^2 = \frac{1}{4}\left[(e^{x} - e^{-x})^2 - (e^{x} + e^{-x})^2\right] \text{ ——— } 5$$

$$= \frac{1}{4}\left[(e^{x} - e^{-x} + e^{x} + e^{-x})(e^{x} - e^{-x} - e^{x} - e^{-x})\right]$$

$$= \frac{1}{4} \times 2e^{x} \times (-2e^{-x})$$

$$= -1$$

$$\left(\frac{dy}{dx}\right)^2 = y^2 - 1 \text{ ——— } 5$$

II
$$\frac{d\left(\frac{\tan x}{1+\sin x}\right)}{dx} = \frac{(1+\sin x)\sec^2 x - \tan x \cos x}{(1+\sin x)^2} \text{ ——— } 5$$

$$= \frac{(1+\sin x) - \cos^2 x \sin x}{\cos^2 x(1+\sin x)^2}$$

$$= \frac{(1+\sin x) - \sin x(1-\sin^2 x)}{\cos^2 x(1+\sin x)^2}$$

$$= \frac{(1+\sin x)(\sin^2 x - \sin x + 1)}{\cos^2 x(1+\sin x)^2}$$

10 $$= \frac{\sin^2 x - \sin x + 1}{\cos^2 x(1+\sin x)} \text{ ——— } 5$$

III. $\dfrac{d(e^t \ln t \cos t)}{dt} = \cos t \times \dfrac{d e^t \ln t}{dt} + e^t \ln t \dfrac{d \cos t}{dt}$

$= \cos t \left[e^t \dfrac{d \ln t}{dt} + \ln t \dfrac{d e^t}{dt} \right] + e^t \ln t \times (-\sin t)$ —— 5

$= \cos t \left[e^t \times \frac{1}{t} + \ln t \cdot e^t \right] - e^t \ln t \times \sin t$

$= e^t \left[\cos t \left(\ln t + \frac{1}{t} \right) - \ln t \sin t \right]$ —— 5

b) $f(x) = \dfrac{x}{(x-1)(x-2)}$

$x = 0$ ஆக $y = 0$

$\therefore$ வளையி $(0, 0)$ இனூடு செல்லும் —— 5

$x = 2, 1$ ஆக $f(x) \longrightarrow \pm \infty$

$\therefore x = 1, x = 2$ நிலைக்குத்து அணுகுகோடுகளாகும் —— 5

$$\left. \begin{aligned} f'(x) &= \frac{(x^2 - 3x + 2) - x(2x - 3)}{(x-1)^2 (x-2)^2} \\ &= \frac{(2 - x^2)}{(x-1)^2 (x-2)^2} \end{aligned} \right\} 5$$

* கிடை அணுகுகோடு

$\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \infty} \dfrac{1/x}{(1 - 1/x)(1 - 2/x)}$

$= 0$

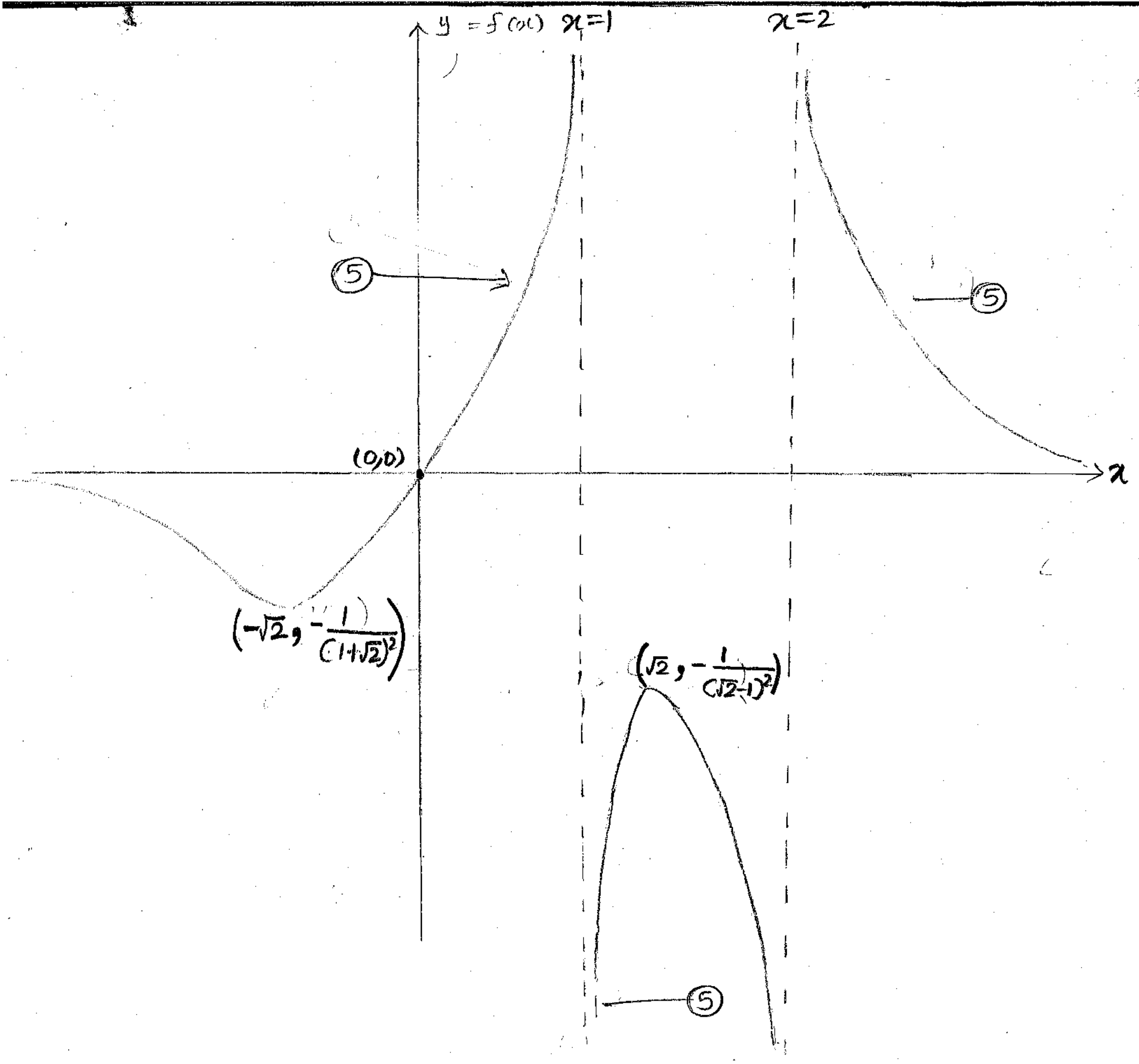
$\therefore$ கிடை அணுகுகோடு $y = 0$. —— 5

$f'(x) = 0$ ஆக $x = \pm \sqrt{2}$

	$-\infty < x < -\sqrt{2}$	$-\sqrt{2} < x < 1$	$1 < x < \sqrt{2}$	$\sqrt{2} < x < 2$	$2 < x < \infty$
$f'(x)$ இன் குறி	$(-)$	$(+)$	$(+)$	$(-)$	$(-)$
$f(x)$	↘	↗	↗	↘	↘
	5	5	5	5	5

$\therefore f(x)$ ஆனது $\left(-\sqrt{2}, \dfrac{-1}{(1+\sqrt{2})^2}\right)$ இல் இழிவடையும் —— 5

$f(x)$ ஆனது $\left(\sqrt{2}, \dfrac{-1}{(\sqrt{2}-1)^2}\right)$ இல் உயர்வடையும் —— 5



$y = f(x)$
$x=1$
$x=2$
x
(0,0)
$\left(-\sqrt{2}, -\frac{1}{(1+\sqrt{2})^2}\right)$
$\left(\sqrt{2}, -\frac{1}{(\sqrt{2}-1)^2}\right)$
5
5
5

$K(x-1)(x-2)-x=0$ [இரு தீர்வு]

$K=\dfrac{x}{(x-1)(x-2)}$

$=f(x)$

$+\infty>k>0$ (5) , $0>k>-\dfrac{1}{(1+\sqrt{2})^2}$ (5) , $-\dfrac{1}{(\sqrt{2}-1)^2}>k>-\infty$ (5)

(c) $AD=2a\cos\theta+a$

$A=\frac{1}{2}\times(2a\cos\theta+a+a)\times a\sin\theta$

$=\frac{1}{2}a^2[2\sin\theta+\sin 2\theta]$ —— (5)

$\dfrac{dA}{d\theta}=\frac{1}{2}a^2[2\cos\theta+2\cos 2\theta]$

$=a^2[\cos\theta+\cos 2\theta]$ —— (5)

$\dfrac{dA}{d\theta}=0$ ஆக $a\neq 0$ —— (5)

$\cos\theta+\cos 2\theta=0$

$2\cos^2\theta+\cos\theta-1=0$

$(2\cos\theta-1)(\cos\theta+1)=0$ —— (5)

$\cos\theta=\frac{1}{2}$ or $\cos\theta=-1$

$\cos\theta=\cos\pi/3$ (பொருத்தமற்றது)

$\theta=\pi/3$ —— (5) $0<\theta<\pi/2$ —— (5)

15) (a) (i)

$t = (1+x)^{1/6}$

$\frac{dt}{dx} = \frac{1}{6}(1+x)^{5/6}$

$dx = 6(1+x)^{5/6}dt$

$= 6t^5 dt$ —— (5)

$\int \frac{x}{\sqrt[3]{1+x} - \sqrt{1+x}}\, dx = \int \frac{x}{[(1+x)^{1/6}]^2 - [(1+x)^{1/6}]^3}\, dx$ —— (5)

$= \int \frac{t^6 - 1}{t^2 - t^3} \cdot 6t^5\, dt$

$= 6\int \frac{(t-1)(t^5+t^4+t^3+t^2+t+1)}{t^2(1-t)}dt$

$= -6\int (t^5+t^4+t^3+t^2+t+1)t^3\, dt$

$= -6\int t^3(t^5+t^3+t+t^4+t^2+1)dt$

$= -6\int (t^8+t^7+t^6+t^5+t^4+t^3)\, dt$

$= -6\left[\frac{t^9}{9} + \frac{t^8}{8} + \frac{t^7}{7} + \frac{t^6}{6} + \frac{t^5}{5} + \frac{t^4}{4}\right] + c$

—— (5)

c-ஒருமை மாறிலி

(ii) $y = (1+x^2)$ என பிரதியிட

$$\left.\begin{aligned}\frac{dy}{dx} &= 2x \\ \frac{1}{2}\,dy &= x\,dx\end{aligned}\right\} \text{ ⑤}$$

$x = 1 \iff y = 2$

$x = 0 \iff y = 1$

$$\int_0^1 \frac{x}{(1+x^2)^{3/2}}\,dx = \int_1^2 \frac{1}{y^{3/2}} \cdot \frac{1}{2}\,dy \text{ ⑤}$$

$$= \frac{1}{2}\int_1^2 y^{-3/2}\,dy$$

$$= \frac{1}{2}\left[\frac{y^{-1/2}}{-1/2}\right]_1^2$$

$$= -1\left[\frac{1}{\sqrt{y}}\right]_1^2$$

$$= -1\left[\frac{1}{\sqrt{2}} - 1\right]$$

$$= 1 - \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(\sqrt{2}-1) \text{ ⑤}$$

(b) $y = a - x$ எனின்

$$\frac{dy}{dx} = (-1)$$

$$dx = -dy$$

$$x = 0 \Leftrightarrow y = a$$
$$x = a \Leftrightarrow y = 0 \quad \text{——— } 5$$

$$\int_0^a f(a-x)\,dx = \int_a^0 f(y)(-dy)$$

$$= \int_0^a f(y)\,dy \quad \text{——— } 5$$

$$= \int_0^a f(x)\,dx$$

$$\int_0^\pi (x \sin^2 x)\,dx = \int_0^\pi (\pi - x)\sin^2(\pi - x)\,dx \quad \text{——— } 5$$

$$= \int_0^\pi (\pi - x)\sin^2 x\,dx$$

$$= \pi\int_0^\pi \sin^2 x\,dx - \int_0^\pi x\sin^2 x\,dx$$

$$\int_0^\pi x\sin^2 x\,dx = \frac{\pi}{2}\int_0^\pi \sin^2 x\,dx \quad \text{——— } 5$$

$$= \frac{\pi}{2}\int_0^\pi \frac{(1-\cos 2x)}{2}\,dx \quad \text{——— } 5$$

$$= \frac{\pi}{4}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$$

$$= \frac{\pi}{4}\left[\pi - 0\right]$$

$$= \frac{\pi^2}{4} \quad \text{——— (5)}$$

(c) $t = \tan\frac{x}{2}$

$$\left.\begin{aligned} x = 0 &\iff t = 0 \\ x = \frac{\pi}{2} &\iff t = 1 \end{aligned}\right\} (5)$$

$$\frac{dt}{dx} = \left(\sec^2\frac{x}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}\left(1 + \tan^2\frac{x}{2}\right)$$

$$= \frac{1}{2}\left(1 + t^2\right)$$

$$dx = \frac{2}{(1+t^2)}\,dt \quad \text{——— (5)}$$

$$\cos x = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2}}$$

$$= \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}} \qquad \left[\cos^2\frac{x}{2} \neq 0\right]$$

$$= \frac{1 - t^2}{1 + t^2} \quad \text{——— (5)}$$

$$\int_0^{\pi/2} \frac{1}{3+5\cos x}\,dx = \int_0^1 \frac{1}{3+\frac{5(1-t^2)}{(1+t^2)}} \cdot \frac{2}{(1+t^2)}\,dt$$

$$= \int_0^1 \frac{2}{3+3t^2+5-5t^2}\,dt \text{ --- } 5$$

$$= \int_0^1 \frac{2}{8-2t^2}\,dt$$

$$= \int_0^1 \frac{1}{4-t^2}\,dt$$

$$= -\int_0^1 \frac{1}{(t-2)(t+2)}\,dt$$

$$= -\frac{1}{4}\int_0^1 \left[\frac{1}{(t-2)} - \frac{1}{(t+2)}\right] dt \text{ --- } 5$$

$$= -\frac{1}{4}\Big[\ln|t-2| - \ln|t+2|\Big]_0^1$$

$$= -\frac{1}{4}\left[\ln\frac{1}{3} - \ln 1\right]$$

$$= -\frac{1}{4}\ln\left(\tfrac{1}{3}\right) \text{ --- } 5$$

(d) (i) $\cos 3x = 4\cos^3 x - 3\cos x$

$$= \frac{\cos 3x + 3\cos x}{4} \text{ --- } ⑤$$

$$\int x\cos^3 x \, dx = \int \frac{x[\cos 3x + 3\cos x]}{4} dx \text{ --- } ⑤$$

$$= \frac{1}{4}\int x\cos 3x \, dx + \frac{3}{4}\int x\cos x \, dx$$

$$= \frac{1}{4}\int x \frac{d}{dx}\left(\frac{\sin 3x}{3}\right) dx + \frac{3}{4}\int x \frac{d\sin x}{dx} \, dx \text{ --- } ⑤$$

$$= \frac{1}{4}\, \frac{x\sin 3x}{3} - \frac{1}{4}\int \frac{\sin 3x}{3} dx + \frac{3}{4} \times x\sin x - \frac{3}{4}\int \sin x \, dx$$

$$= \frac{x\sin 3x}{12} + \frac{1}{12} \times \frac{\cos 3x}{3} + \frac{3x\sin x}{4} + \frac{3\cos x}{4} \text{ --- } ⑤$$

(ii) $\int \sin x \ln(\sin x) dx = \int \ln \sin x \frac{d}{dx}(-\cos x) \, dx$

$$= -\sin x\cos x + \int \left[\cos x \frac{d}{dx}\ln \sin x\right] dx \text{ --- } ⑤$$

$$= -\frac{\sin 2x}{2} + \int \cos x \times \frac{1}{\sin x} \times \cos x \, dx$$

$$= -\frac{\sin 2x}{2} + \int \frac{(1-\sin^2 x)}{\sin x} dx$$

$$= -\frac{\sin 2x}{2} + \int \frac{\operatorname{cosec} x[\operatorname{cosec} x + \cot x]}{\operatorname{cosec} x + \cot x} dx$$

$$- \int \sin x \, dx$$

$$= -\frac{\sin 2x}{2} - \ln|\operatorname{cosec} x + \cot x| + \cos x + c \quad \text{---} (5)$$

C- ஏதேச்சை மாறிலி

(e) $$\frac{2-x+x^2}{(1+x)(1-x)^2} = \frac{A}{(1+x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2}$$

$$(x^2 - x + 2) = A(1-x)^2 + B(1+x)(1-x) + C(1+x)$$
$$= A(x^2 + 1 - 2x) + B(1 - x^2) + C(1+x)$$

$$\left.\begin{array}{lll} x^0 \Rightarrow & 2 = A+B+C & \text{---} (1) \\ x^2 \Rightarrow & 1 = A-B & \text{---} (2) \\ x \Rightarrow & -1 = -2A+C & \text{---} (3) \\ (1)-(3) \Rightarrow & 3 = 3A+B & \text{---} (4) \end{array}\right\} (5)$$

$$\left.\begin{array}{l} (2)+(4) \Rightarrow 4 = 4A \\ A = 1 \\ B = 0 \\ C = 1 \end{array}\right\} \text{---} (10)$$

$$\frac{2-x+x^2}{4x(1-x)^2} = \frac{1}{(1+x)} + \frac{1}{(1-x)^2} \quad \text{---} (5)$$

$$\int_0^{1/2} \frac{2-x+x^2}{(1+x)(1-x)^2}\,dx = \int_0^{1/2} \left[\frac{1}{(1+x)} + \frac{1}{(1-x)^2}\right] dx \quad \text{---} (5)$$

$$= \int_0^{1/2} \frac{1}{(1+x)}\,dx - \int_0^{1/2} \frac{1}{(1-x)^2}\,d(1-x)$$

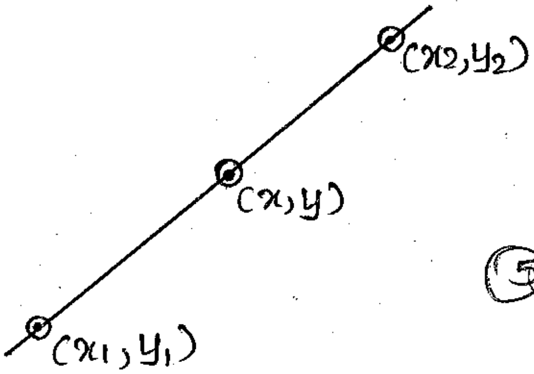
$$= \left[\ln|1+x|\right]_0^{1/2} - \left[\frac{(1-x)^{-1}}{-1}\right]_0^{1/2}$$

$$= \ln\left(\tfrac{3}{2}\right) + \left[\frac{1}{1-x}\right]_0^{1/2}$$

$$= \ln\left(\tfrac{3}{2}\right) + \left[\frac{1}{1/2} - 1\right] \quad 5$$

$$= \ln\left(\tfrac{3}{2}\right) + 1$$

16) a)



$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{y - y_1}{x - x_1} \text{ — } 5$$

$$5 \text{ — } \frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2} = \gamma \text{ என்க}$$

$$\frac{y - y_1}{y_1 - y_2} = \gamma$$

$$y = \gamma y_1 + (1-\gamma) y_2 \text{ — } 5$$

——— R_1

$$\frac{x - x_1}{x_1 - x_2} = \gamma$$

$$x = \gamma x_1 + (1-\gamma) x_2 \text{ ——— } R_2 \text{ — } 5$$

$P \equiv (-2, 3)$
$Q \equiv (2, 3)$ என்க

$x_1 = -2 \quad x_2 = 2$
$y_1 = 3 \quad y_2 = 3$ } என R_1, R_2 இல் பிரதியிட — 10

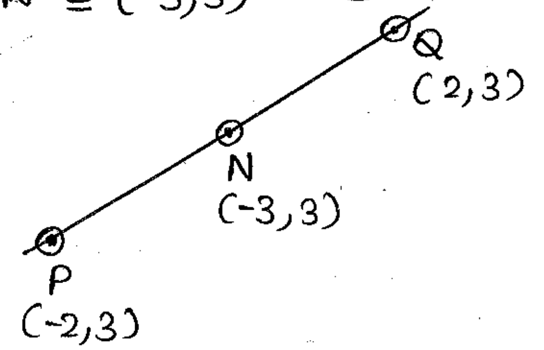
$$y = 3\gamma + (1-\gamma)3$$
$$= 3 \text{ — } 5$$
$$x = \gamma(-2) + (1-\gamma)2$$
$$= 2 - 4\gamma \text{ — } 5$$

x, y என்பன $x + y = 0$ இல் அமைந்திருப்பதால்,

$$2 - 4\gamma + 3 = 0$$
$$\gamma = 5/4 \text{ — } 10$$

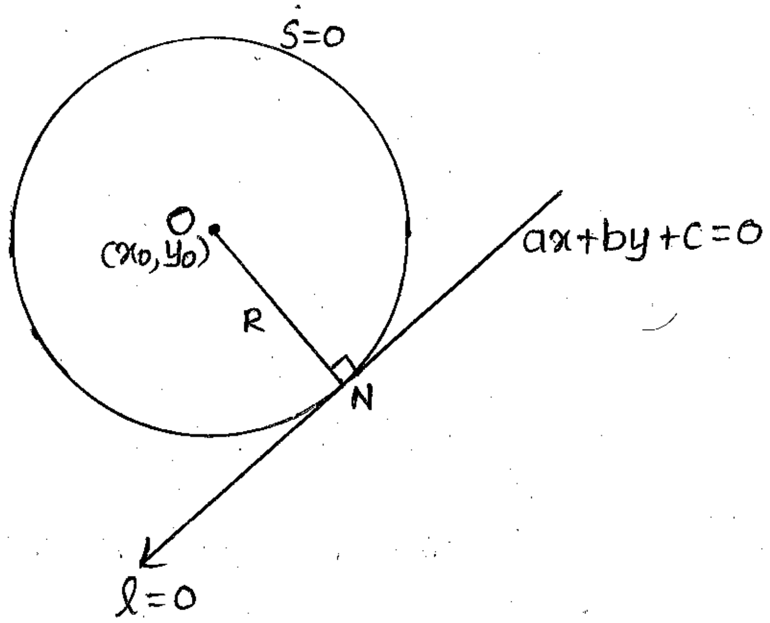
16

$N \equiv (-3,3)$ —— 5



$\dfrac{PN}{NQ} = \dfrac{1}{5}$ —— 5

b)



$ON \perp l=0$

$$\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}} = R \quad \text{—— 5}$$

$$(ax_0+by_0+c)^2 = R^2(a^2+b^2) \text{ ——— } (R_3) \text{ —— } 5$$

$(R_3) \Rightarrow$ இல் $R=1$, $a=4$, $b=-3$, $c=0$ எனப்பிரதியிட

$$(4x_0 - 3y_0)^2 = 25 \text{ ——— } (1) \text{ —— } 5$$

வட்டம் x அச்சைத் தொடுவதால், ⑤

$y_0 = R = 1$

$$(4x_0-3)^2 = 25$$
$$\left.\begin{aligned}16x_0^2-24x_0+9 &= 25\\ 16x_0^2-24x_0-16 &= 0\\ 2x_0^2-3x_0-2 &= 0\end{aligned}\right\} ⑤$$
$$(x_0-2)(2x_0+1)=0$$

$x_0 = 2$ ⑤ $\quad 2x_0+1 \neq 0$ ∵ [வட்டத்தின் மையம் 1ம் கால்வட்டத்தில் இருப்பதால் $x_0>0$] ⑤

∴ மையம் $O \equiv (2,1)$ ⑤

$$S_1 \equiv (x-2)^2+(y-1)^2 = 1$$
$$x^2+y^2-4x-2y+4=0 \quad ⑤$$

(R3) இல் L.H.S இல் $a=3, b=4, c=-15$ $x_0=2$ $y_0=-1$

$L.H.S = (6+4-15)^2$
$=25$

$RHS = 1(9+16) = 25$

$L.H.S = R.H.S$ ⑤

∴ S_1 ஆனது $3x+4y=15$ எனும் கோட்டை தொடும்.

S_2 இன் மையம் $N \equiv (x_1, y_1)$ என்க

(R3) இல் $a=4$, $b=-3$, $c=0$ ஆக

$$(4x_1-3y_1)^2 = 25R^2 \quad (1) \quad ⑤$$

வட்டம் x அச்சைத் தொடுவதால்,

$$y_1 = R \quad (2)$$

(R3) இல் $a=3$, $b=4$, $c=15$ ஆக

$$(3x_1+4y_1-15)^2 = 25R^2 \quad (3) \quad ⑤$$

$$① \Rightarrow (4x_1 - 3R)^2 = (5R)^2$$
$$4x_1 - 3R = 5R \quad (x_1 > 0)$$
$$x_1 = 2R \quad ⑤$$

$$③ \Rightarrow (6R + 4R - 15)^2 = 25R^2$$
$$25(2R-3)^2 = 25R^2$$
$$(2R - 3) = \pm R \quad ⑤$$

$$⊕ \Rightarrow R = 3$$
$$x_1 = 6$$
$$y_1 = 3 \quad ⑤$$

$$⊖ \Rightarrow R = 1 \quad [\text{வட்டம் } S_1 = 0]$$

$$\therefore S_2 \equiv (6-x)^2 + (3-y)^2 = 9$$
$$x^2+y^2-12x-6y+36=0 \quad ⑤$$

$S_1 = 0$, $S_2 = 0$ இன் மையங்களை விட்டமாக உடைய வட்டம் $S_3 = 0$ என்க.

$$S_3 \equiv \frac{(y-3)}{(x-6)} \times \frac{(y-1)}{(x-2)} = (-1) \quad ⑤$$
$$(y^2-4y+3) = -1(x^2-8x+12)$$
$$x^2+y^2-8x-4y+15=0 \quad ⑤$$

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17) (a) (i) L.H.S $= \tan\alpha - 2\tan(\alpha + \pi/4) + \tan(\alpha + \pi/2)$

$$= \tan\alpha - 2\left[\frac{\tan\alpha + \tan\pi/4}{1 - \tan\alpha\tan\pi/4}\right] - \frac{1}{\tan\alpha} \quad \text{—— } 5$$

$$= \frac{\tan^2\alpha - 1}{\tan\alpha} - \frac{2(\tan\alpha + 1)}{(1 - \tan\alpha)}$$

$$= \frac{(\tan\alpha - 1)(\tan\alpha + 1)}{\tan\alpha} + \frac{2(\tan\alpha + 1)}{(\tan\alpha - 1)}$$

$$= \frac{(\tan\alpha + 1)(\tan^2\alpha - 2\tan\alpha + 1 + 2\tan\alpha)}{\tan\alpha(\tan\alpha - 1)} \quad \text{—— } 5$$

$$= \frac{(\tan\alpha + 1)(\tan^2\alpha + 1)}{\tan\alpha(\tan\alpha - 1)}$$

= R.H.S

(ii) $\tan\theta = \frac{4}{3}$ $\quad 0 < \theta < 2\pi$

$$\frac{2\tan\theta/2}{1 - \tan^2\theta/2} = \frac{4}{3} \quad \text{—— } 5$$

$\tan\theta/2 = a$ எனின்

$$\frac{2a}{1 - a^2} = \frac{4}{3} \quad \text{—— } 5$$

$$6a = 4 - 4a^2$$

$$2a^2 + 3a - 2 = 0$$

$$(2a - 1)(a + 2) = 0$$

$a = 1/2$ or $a = -2$ —— 5

18

$\tan\frac{\theta}{2} = \frac{1}{2}$ or $\tan\frac{\theta}{2} = -2$

ஆனால், $0 < \theta < 2\pi$

$0 < \theta/2 < \pi$

$\therefore$ இவ்விரண்டு பெறுமானமும் பொருந்தும் — 5

$\tan\frac{\theta}{2} = \frac{1}{2}$ எனின் $\sin\theta/2 = 1/\sqrt{5}$ — 5

$\tan\frac{\theta}{2} = -2$ எனின் $\sin\theta/2 = 2/\sqrt{5}$

---

b) (i) $L \cdot H \cdot S = 4\sin(60-\theta)\sin\theta\sin(60+\theta)$

$= 2\sin\theta\left[2\sin(60-\theta)\sin(60+\theta)\right]$ — 5

$= 2\sin\theta\left[\cos 2\theta - \cos\frac{2\pi}{3}\right]$

$= 2\sin\theta\left[\cos 2\theta + \frac{1}{2}\right]$

$= 2\sin\theta\left[1 - 2\sin^2\theta + \frac{1}{2}\right]$ — 5

$= 2\sin\theta - 4\sin^3\theta + \sin\theta$

$= 3\sin\theta - 4\sin^3\theta$ — 5

$= \sin 3\theta$

$= R \cdot H \cdot S$

$\theta = 20$ என பகுதி (i) இல் பிரதியிட

$4\sin 40\sin 20\sin 80 = \frac{1}{4}\sin^2 60$ — 5

$= \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{16}$ — 5

(ii) $R.H.S = -8\sin\theta \sin(\theta-\pi/4)\sin(\theta-\pi/2)\sin(\theta-3\pi/4)$

$= -4\sin\theta \sin(\theta-\pi/2)\left[2\sin(\theta-\pi/4)\sin(\theta-3\pi/4)\right]$

$= -4\sin\theta(-\cos\theta)[\cos\pi/2 - \cos(2\theta-\pi)]$ — 5

5

$= 4\sin\theta\cos\theta\,[0 - \cos(\pi-2\theta)]$

$= 4\sin\theta\cos\theta\,[\cos 2\theta]$ — 5

$= 2\sin 2\theta \cos 2\theta$

$= \sin 4\theta$

(iii) $L.H.S = \sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)$

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 2\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\sin\left(\frac{-\alpha-\beta}{2}\right)$ — 5

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$ 5

$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[2\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\right]$

$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{\gamma+\beta}{2}\right)$ — 5

(c)

$$\left.\begin{aligned} \tan^{-1}(1/7) &= A \\ \tan^{-1}(1/13) &= B \\ \tan^{-1}(2/9) &= C \end{aligned}\right\} \text{எனின்} \quad 5$$

$\therefore \tan A = 1/7$

$\tan B = 1/13$

$\tan C = 2/9$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad 5$$

$$= \frac{\frac{1}{7} + \frac{1}{13}}{1 - \left(\frac{1}{7}\right)\left(\frac{1}{13}\right)} \quad 5$$

$$= \frac{20}{90}$$

$$= \frac{2}{9}$$

$$= \tan C \quad 5$$

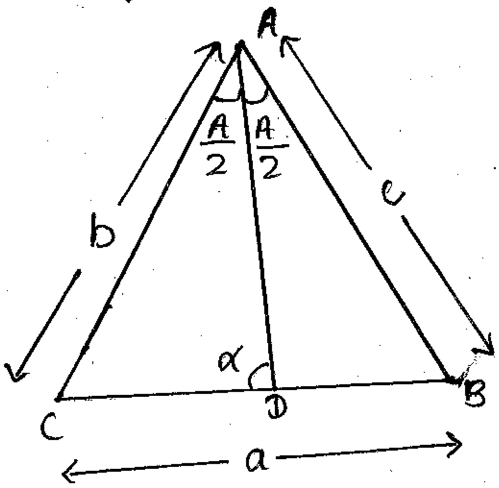
$\therefore A + B = C$ — 5

$$\tan^{-1}(1/7) + \tan^{-1}(1/13) = \tan^{-1}(2/9)$$

(d) ΔACD இல் Sin விதிப்படி

$$\frac{b}{\sin\alpha} = \frac{CD}{\sin(A/2)} \quad \text{—— } 5$$

$$CD = \frac{b\sin(A/2)}{\sin\alpha} \quad \text{—— } ①$$



ΔABD இல் Sin விதிப்படி

$$\frac{c}{\sin(\pi-\alpha)} = \frac{BD}{\sin A/2} \quad \text{—— } 5$$

$$BD = \frac{c\sin A/2}{\sin\alpha} \quad \text{—— } ②$$

ΔABD இல்

$$\frac{AD}{\sin B} = \frac{c}{\sin(\pi-\alpha)} \quad \text{—— } 5$$

$$\sin\alpha = \frac{c\sin B}{AD} \quad \text{—— } ③$$

$$① + ② \Rightarrow CD + BD = \frac{(b+c)\sin(A/2)}{\sin\alpha}$$

$$a\sin\alpha = (b+c)\sin(A/2) \quad \text{—— } 5$$

$$\frac{ac\sin B}{AD} = (b+c)\sin(A/2) \quad \because ③$$

$$AD(b+c)\sin A/2 = ac\sin B$$

$$AD(b+c)\sin A/2 = b\sin A \cdot c$$

$$AD(b+c)\sin A/2 = 2bc\sin A/2\cos A/2 \quad \text{—— } 5$$

$$AD(b+c) = 2bc\cos A/2 \quad \text{—— } R_1$$

ΔABC இல் cos விதிப்படி

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = b^2 + c^2 - 2bc\left[2\cos^2 A/2 - 1\right]$$
$$a^2 = (b+c)^2 - 4bc\cos^2(A/2) \text{ ------ } (4) \text{ --- } (5)$$

$$(R_1)^2 \Rightarrow \quad AD^2(b+c)^2 = 4(bc)^2\cos^2 A/2$$
$$4bc\cos^2 A/2 = \frac{AD^2(b+c)^2}{bc} \text{ --- } (5)$$

$$(4) \Rightarrow \quad a^2 = (b+c)^2 - \frac{AD^2(b+c)^2}{bc}$$
$$a^2 = (b+c)^2\left[1 - \frac{AD^2}{bc}\right]$$
$$a = (b+c)\left[1 - \frac{AD^2}{bc}\right]^{1/2} \text{ ------ } (R_2) \text{ --- } (5)$$

$$\left.\begin{array}{l} AB = 9 \\ AC = 5 \\ AD = 6 \end{array}\right\} \text{எனின்}$$

$c = 9$
$b = 5$

$$(R_1) \Rightarrow \quad 6(5+9) = 2\times 5\times 9\cos(A/2)$$
$$\cos A/2 = \frac{14}{15}$$
$$A/2 = \cos^{-1}\left(\frac{14}{15}\right)$$
$$A = 2\cos^{-1}\left(\frac{14}{15}\right) \text{ ------ } (5)$$

$$(R_1) \quad 6(5+9) = 2\times5\times9 \cos\left(A/2\right)$$

$$\cos(A/2) = \frac{14}{15}$$

$$A/2 = \cos^{-1}(14/15)$$

$$A = 2\cos^{-1}\left(\frac{14}{15}\right)$$

$R_2 \Rightarrow$ $$a = (5+9)\left[1 - \frac{b^2}{5 \times 9}\right]^{1/2}$$

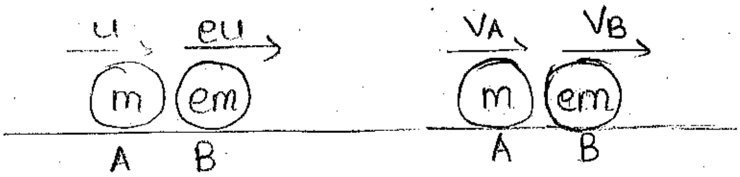
$$a = 14\left[1 - \frac{36}{45}\right]^{1/2}$$

$$\left.\begin{aligned} a &= \frac{14}{\sqrt{5}} \\ &= \frac{14\sqrt{5}}{5} \end{aligned}\right\} 5$$

$\boxed{150}$

பகுதி A

01)



தொகுதிக்கு $\overrightarrow{I} = \Delta mv$

$$(mu + me^2u) - (mV_A + emV_B) = 0$$

$$V_A + eV_B = u(1+e^2) \text{ ——— } (1) \quad 5$$

நியூட்டனின் பரிசோதனை விதிப்படி

$$e = \frac{V_B - V_A}{u - eu}$$

$$V_B - V_A = eu(1-e) \text{ ——— } (2) \quad 5$$

$$(1)+(2) \Rightarrow (e+1)V_B = (1+e)u$$

$$V_B = u \quad 5 \quad (\because e > 0)$$

B க்கு $\overrightarrow{I} = \Delta mV$

$$em(V_B - eu) = \frac{6mu}{25} \quad 5$$

$$25e^2 - 25e + 6 = 0$$

$$(5e-3)(5e-2) = 0$$

$\therefore\ e = 3/5$ or $e = 2/5$ 5

$\boxed{25}$

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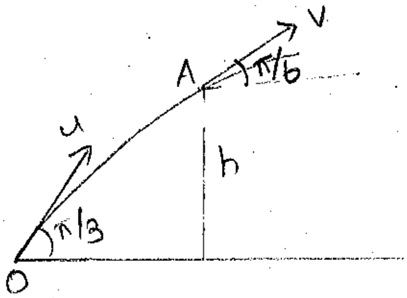
02)



எல்லாப்புள்ளிகளிலும் கிடைவேகம் சமன்.

$$\therefore\ u\cos\pi/3 = v\cos\pi/6 \quad 5$$

$$u/2 = v \cdot \sqrt{3}/2$$

$$\therefore\ v = u/\sqrt{3} \quad 5$$

A இல் நிலைக்குத்து வேகம்

$$= v\sin\pi/6$$

$$= \frac{u}{\sqrt{3}} \cdot \frac{1}{2}$$

$$= \frac{u}{2\sqrt{3}} \quad 5$$

O இலிருந்து A வரை

$$\uparrow\ v^2 = u^2 + 2as$$

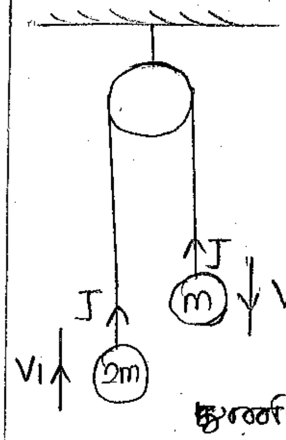
$$\left(\frac{u}{2\sqrt{3}}\right)^2 = (u\sin\pi/3)^2 - 2gh \quad 5$$

$$\frac{u^2}{12} = \frac{3u^2}{4} - 2gh$$

$$h = \frac{u^2}{3g} \quad 5$$

$\boxed{25}$

03)



துணிக்கை m க்கு ⇒

$\downarrow v^2 = u^2 + 2as$ (5)

$v^2 = 0 + 2gh$

$v = \sqrt{2gh}$ (5)

துணிக்கை m ற்கு ⇒ $\downarrow I = \Delta mv$

$-J = m(V_1 - v)$ (5) —— (1)

துணிக்கை 2m ற்கு ⇒ $\uparrow I = \Delta mv$

$J = 2m(V_1 - 0)$ (5) —— (2)

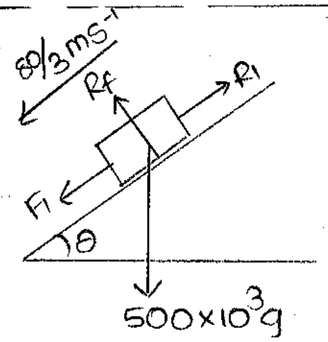
(1) + (2) ⇒ $3mV_1 - mV = 0$

$V_1 = V/3$

$\therefore V_1 = \frac{1}{3}\sqrt{2gh}$ (5)

25

04)



$96\,kmh^{-1}$

$= \frac{96 \times 10^3 m}{3600 s}$

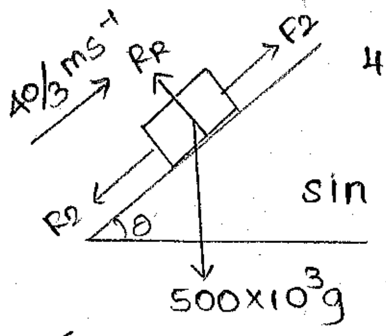
$= 80/3\, ms^{-1}$

$\overleftarrow{F} = ma$

$F_1 + 500 \times 10^4 \times \sin\theta - R_1 = 0$

$R_1 = F_1 + 5 \times 10^4$ —— (1)

(5)



$48\,kmh^{-1}$

$= 40/3\, ms^{-1}$

$\sin\theta = 1/100$

$\overrightarrow{F} = ma$

$F_2 - R_2 - 500 \times 10^4 \sin\theta = 0$

$R_2 = F_2 - 5 \times 10^4$ —— (2)

(5)

$R \propto V$ (5)

$R_1 \propto 80/3$ —— (3)

$R_2 \propto 40/3$ —— (4)

$P = F \cdot V$ (5)

$P = F_1 \times 80/3$ —— (5)

$P = F_2 \times 40/3$ —— (6)

(3)/(4) ⇒ $R_1/R_2 = 2$

(5), (6) ⇒ $2F_1 = F_2$

(1)/(2) ⇒ $\frac{F_1 + 5 \times 10^4}{F_2 - 5 \times 10^4} = 2$

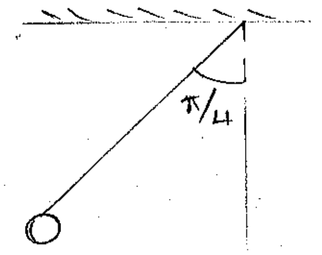
$\therefore F_1 = 5 \times 10^4$

(5) ⇒ $P = 5 \times 10^4 \times 80/3$ (5)

$= 1333\,kW$

25

05)



பொறிமுறை சக்திக் காப்பு விதிப்படி 5

$2mg(l - l\cos\pi/4) + 0 = \frac{1}{2}(2m)v^2$ 5

$2gl\left(1 - \frac{1}{\sqrt{2}}\right) = v^2$

$v^2 = gl(2-\sqrt{2})$ 5

$v = \sqrt{gl(2-\sqrt{2})}$

M ZPL

$\overrightarrow{I = \Delta mv}$

$2mv + 0 = 3mv'$ 5

$v' = 2v/3$

$= \frac{2}{3}\sqrt{lg(2-\sqrt{2})}$ 5

$\boxed{25}$

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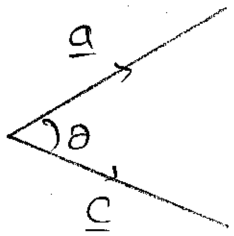
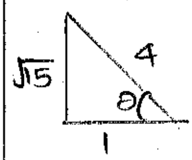
06)



$\tan\theta = \sqrt{15}$



$b - 2a = k\underline{c}$

$\underline{b} = 2\underline{a} + k\underline{c}$

$\underline{b}\cdot\underline{b} = (2\underline{a} + k\underline{c})(2\underline{a} + k\underline{c})$ 5

$|\underline{b}|^2 = 4|\underline{a}|^2 + k^2|c|^2 + 4k\,\underline{a}\cdot\underline{c}$ 5

$16 = 4 + k^2 + 4k|a||c|\cos\theta$ 5

$12 = k^2 + 4k \times 1 \times 1 \times \frac{1}{4}$

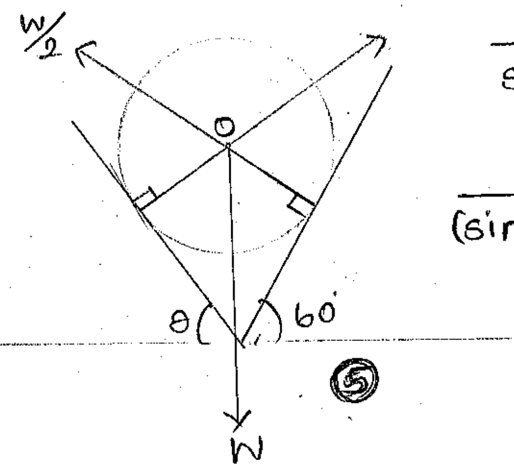
$k^2 + k - 12 = 0$ 5

$(k+4)(k-3) = 0$

$k = -4$ or $k = 3$ 5

$\boxed{25}$

07)



5

O இல் லாமியின் தேற்றப்படி

$$\frac{W}{\sin(\theta+60^\circ)} = \frac{W/2}{\sin(180-\theta)} \quad 5$$

$$\frac{1}{(\sin\theta\cos 60+\cos\theta\sin 60)} = \frac{1}{2\sin\theta} \quad 5$$

$$1+\cot\theta \times \sqrt{3} = 4$$

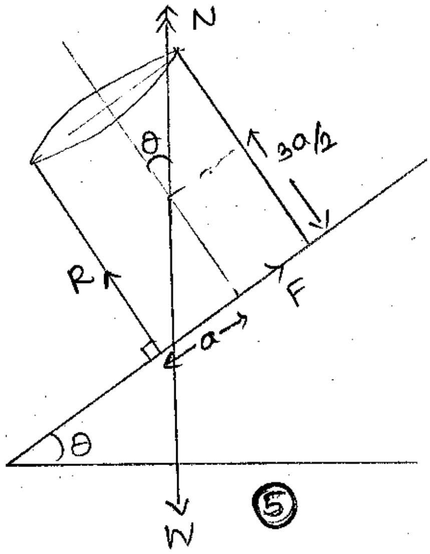
$$\cot\theta = \sqrt{3} \quad 5$$

$$\theta = \pi/6 \quad (0<\theta<\pi/2)$$

5

$\boxed{25}$

08)



5

சமநிலைக்கு

$\theta \le \lambda$ ஆக வேண்டும். 5

λ - உராய்வுக் கோணம்.

$$\tan\lambda = \mu$$

$$\tan\theta = \frac{a}{3a/2} = 2/3 \quad 5$$

$$\tan\theta \le \tan\lambda \quad 5$$

$$2/3 \le \mu$$

$\therefore \mu < 2/3$ எனின் சமநிலை குழம்பும். 5

$\boxed{25}$

09) A - குடிசார் பொறியியல் கற்றவர் 19

B - ஆணாக இருத்தல் 13

i) $P(A) = 19/30$ 10

ii) $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ 5

$= \dfrac{7/30}{13/30}$ 5

$= 7/13$ 5

$\boxed{25}$

---

10) $S_x^2 = \dfrac{\sum_{r=1}^{n} x_i^2}{n} - (\bar{x})^2$ 5

$36 = \dfrac{1620}{n} - \left(\dfrac{\sum_{r=1}^{n} x_i}{n}\right)^2$ 5

$36 = \dfrac{1620}{n} - \left(\dfrac{108}{n}\right)^2$ 5

$1 = \dfrac{45}{n} - \dfrac{324}{n^2}$

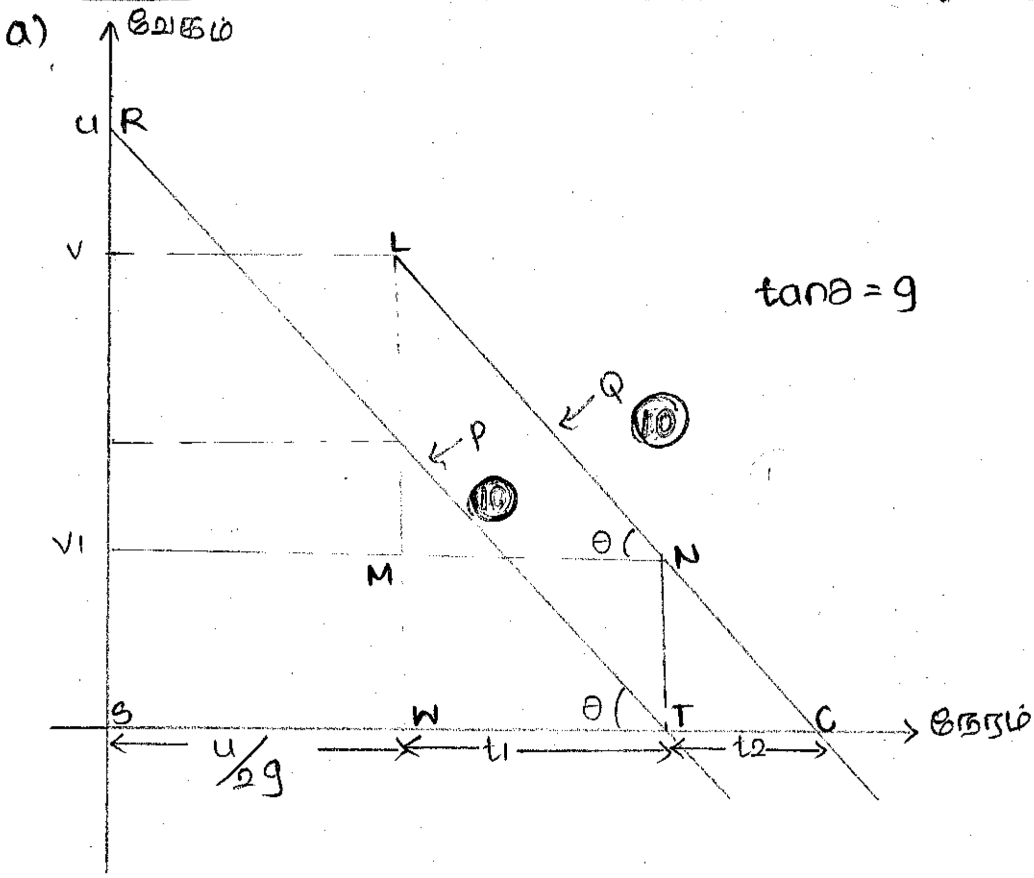
$n^2 - 45n + 324 = 0$ 5

$(n-36)(n-9) = 0$

$n = 36$ or $n = 9$ 5

$\boxed{25}$

11) a)



$\tan\theta = g$

(i) ΔRST இல்

$$g = \frac{u}{\left(\frac{u}{2g} + t_1\right)}$$

$$t_1 = \frac{u}{2g} \quad 5$$

ΔLMN இல்

$$g = \frac{V - V_1}{t_1}$$

$$V - V_1 = \frac{u}{2} \quad 5$$

$$V_1 = \left(V - \frac{u}{2}\right)$$

$$\left(V > \frac{u}{2}\right)$$

(ii) $V = \frac{11u}{12}$

$$V_1 = \frac{11u}{12} - \frac{u}{2} = \frac{5u}{12} \quad 5$$

$AB = OA - OB$ 5

$= \Delta RST$ பரப்பு − சரிவகம் $LMWTN$ பரப்பு

$$= \frac{1}{2} \times \frac{u}{g} \times u \ (5) - \frac{1}{2} \times \left(\frac{11u}{12} + \frac{5u}{12}\right) \times \frac{u}{2g} \ (5)$$

$$= \frac{u^2}{6g} \quad 5$$

iii) ΔNTC யில்

$$g = \frac{v_1}{t_2}$$

$$t_2 = \frac{5u}{12g} \quad 5$$

$$\Delta NTC \equiv \Delta TCD$$

$$\Delta TCD \text{ பரப்பு} = \frac{1}{2} \times \frac{5u}{12g} \times \frac{5u}{12} \quad 5$$

$$= \frac{25u^2}{288g} \quad 5$$

Q வில் அதியுயர் புள்ளியை அடைகையில் P யின் உயரம்

$$= \frac{u^2}{2g} - \frac{25u^2}{288g} \quad 5$$

$$= \frac{119u^2}{288g} \quad 5$$

$\boxed{80}$

b) S - போர்க்கப்பல் B - படகு

$$V_{SE} = u$$

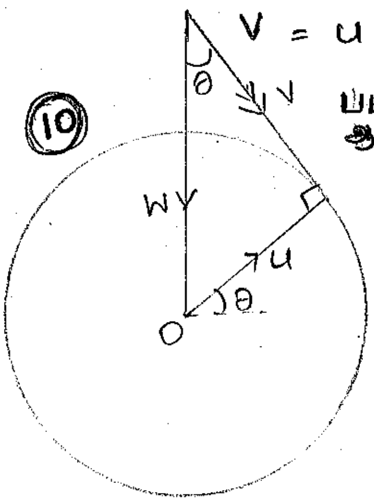
$$V_{BE} = \uparrow w$$

சார்பு வேகக் கோட்பாட்டின் படி ⇒

$$V_{SB} = V_{SE} + V_{EB} \quad 5$$

$$= V_{SE} - V_{BE}$$

$$V = u + \downarrow w \quad 5$$

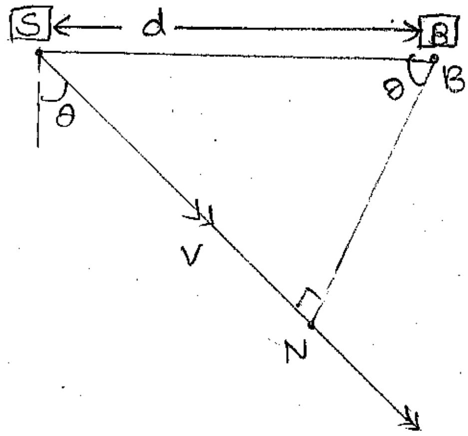


படகு சார்பான போர்க்கப்பலின் பாதை நிலைக்குத்துடன் ஆக்கும் கோணம் உயர்வாக இருக்க 5

$$\sin\theta = \frac{u}{w}$$

$$\theta = \sin^{-1}\left(\frac{u}{w}\right) \quad 5$$

B இன் சட்டத்தில் S இன் பயணப்பாதை ⇒

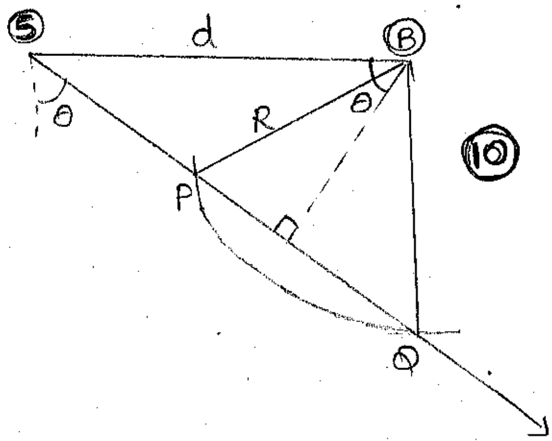


படகு பாதுகாப்பாக செல்வதற்கு ⇒ $R < B_1N$ 10

$R < d\cos\theta$

$R < \dfrac{d\sqrt{w^2-u^2}}{w}$ $[w > u]$

$R > \dfrac{d\sqrt{w^2-u^2}}{w}$ எனின் படகு தாக்கப்படும். 5



10

படகு தாக்கப்படும் நேரம் $= PQ/V$ 5

$$= \frac{2\sqrt{R^2 - d^2\cos^2\theta}}{\sqrt{w^2-u^2}}$$

$$= \frac{2\sqrt{R^2 - d^2\left(\frac{w^2-u^2}{w^2}\right)}}{\sqrt{w^2-u^2}}$$

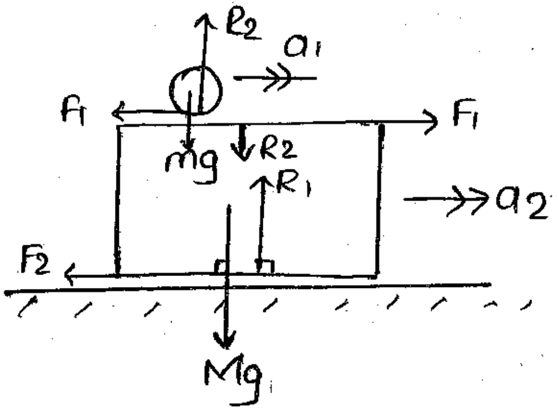
10

$$= \frac{2\sqrt{R^2w^2 - d^2w^2 + d^2u^2}}{w\sqrt{w^2-u^2}}$$

$[w > u]$

70

12)



விசைப்படம் — 5

ஆர்முடுகல்படம் — 5

(m) $\uparrow R_2 = mg$ 5

(M) $\uparrow R_1 - R_2 - Mg = 0$

$R_1 = (M+m)g$ 5

$\leftarrow \quad F_2 = \mu_1 R_1$

$= \mu_1 (M+m) g$ 5

$\rightarrow \quad F_1 = \mu_2 R_2$

$= \mu_2 mg$ 5

(i) (M) ஆனது $\overrightarrow{BC}$ இயங்குவதால்,

$F_1 > F_2$ 5

$\mu_2 mg > \mu_1 (M+m) g$

$m\mu_2 > (m+M)\mu_1$ 5

$a_{m,M} = \overrightarrow{a_2}$

$a_{M,E} = \overrightarrow{a_1}$

$a_{m,E} = a_{m,M} + a_{M,E}$

$= \overrightarrow{a_1} + \overrightarrow{a_2}$

5

(ii) (M) $\rightarrow \quad F = ma$

$F_1 - F_2 = Ma_2$ 5

$\mu_2 mg - \mu_1 (M+m) g = Ma_2$

$$a_2 = \frac{[(\mu_2 - \mu_1)m - \mu_1 M]g}{M}$$ 5

$(m) \rightarrow F = ma$

$$-F_1 = m(a_1 + a_2) \quad 5$$

$$-\mu_2 mg = m(a_1 + a_2)$$

$$(a_1 + a_2) = -\mu_2 g$$

$$a_1 = -\mu_2 g - a_2$$

M // இன் சடடத்தில் 5

$(m) \longrightarrow s = ut + \frac{1}{2} a_1 t^2$

$$x = ut - \frac{1}{2}\left\{\mu_2 g + \frac{[(\mu_2 - \mu_1)m - \mu_1 M]g}{M}\right\}t^2$$

$$Mx = Mut - \frac{1}{2}(\mu_2 - \mu_1)(M+m)g\,t^2 \quad 5$$

$(M) \rightarrow s = ut + \frac{1}{2}at^2$

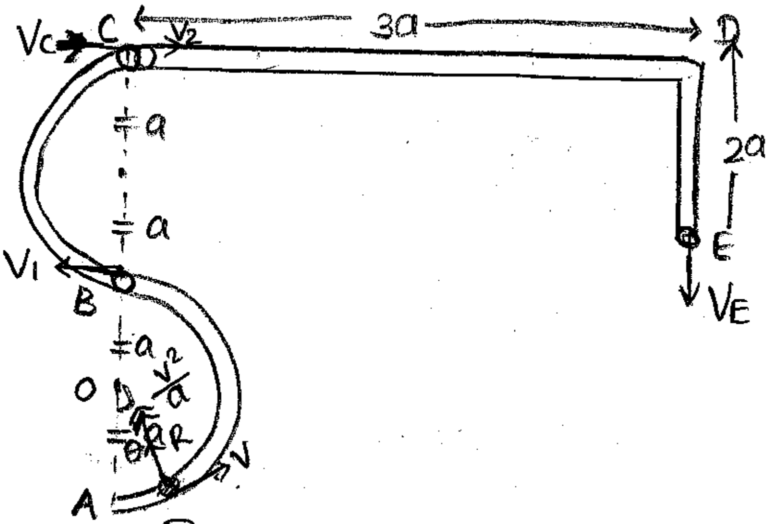
$$y = 0 + \frac{1}{2}a_2 t^2$$

$$y = \frac{1}{2}t^2\left[\frac{(\mu_2 - \mu_1)m - \mu_1 M}{M}\right]g$$

$$\therefore My = \frac{1}{2}[(\mu_2 - \mu_1)m - M\mu_1]g t^2 \quad 5$$

$\boxed{75}$

b)

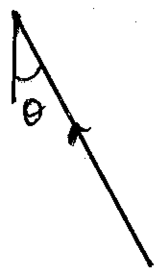


பொறிமுறைசக்திக்காப்புகோட்பாட்டின் படி;

$$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mg(a - a\cos\theta) \quad (5)$$

$$u^2 = v^2 + 2ag(1-\cos\theta)$$

$$v^2 = u^2 - 2ag(1-\cos\theta) \quad (5) \text{———} (1)$$



$F = ma$

$$R - mg\cos\theta = \frac{mv^2}{a} \quad (5)$$

$$R - mg\cos\theta = \frac{m}{a}(u^2 - 2ag(1-\cos\theta))$$

$$R = \frac{m}{a}\left[u^2 - (2-3\cos\theta)ag\right] \quad (5) \text{———} (2)$$

(1) இல் $\theta = \pi$ (5) ஆக $V_1^2 = u^2 - 4ag$ (5)

$u^2 < 4ag$

$V^2 < 0$ (5) ∴ துணிக்கை BC இனூடு செல்ல இயலாது.

$u = \sqrt{12ag}$ ∴ துணிக்கை BC இனூடு செல்லும்.
(5)

$\therefore V_1^2 = 8ag$

$V_1 = \sqrt{8ag}$ (5)

① இல் $u \longrightarrow V_1$

$\theta \longrightarrow \pi$

$V \longrightarrow V_C$ ஆக

$$V_C^2 = 8ag - 4ag$$
$$= 4ag$$

$$V_C = \sqrt{4ag} \quad ⑤$$

C இல் $\rightarrow$ $I = \Delta mv$

$$m\sqrt{4ag} + 0 = 2mV_2$$
$$V_2 = \sqrt{ag} \quad ⑤$$

சேர்த்தித் துணிக்கை CD இனூடு d தூரம் இயங்கியது எனின்.

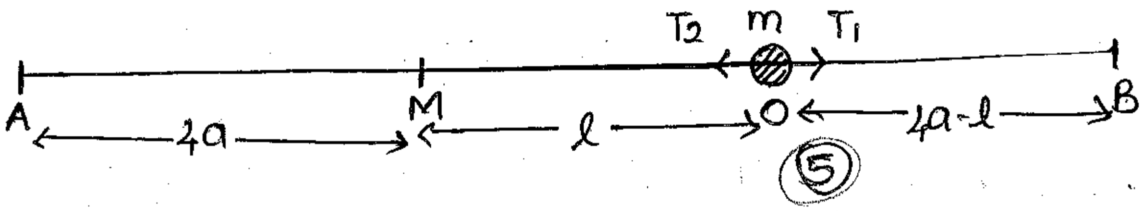
C $\longrightarrow$ D இயக்கசக்தி நட்டம் = உராய்வுக்கு எதிரான வேலை ⑤

$$\frac{1}{2}(2m)V_2^2 = 2mg \times \frac{1}{6}d \quad ⑤$$
$$ag = \frac{1}{3}dg$$
$$3a = d \quad ⑤$$

D இல் $\downarrow v^2 = u^2 + 2as$

$$V_E^2 = 2g(2a)$$
$$V_E = \sqrt{4ag} \quad ⑤$$

13)



சமநிலையில்,

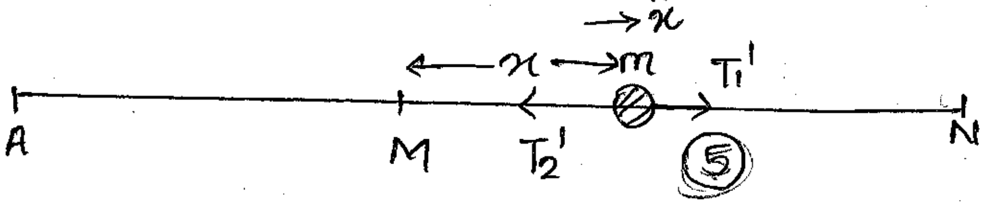
$$T_1 = T_2 \quad 5$$

$$\frac{4\lambda}{3a}(a-l) = \frac{\lambda}{2a}(2a+l) \quad 5$$

$$8a - 8l = 6a + 3l$$

$$l = \frac{2a}{11}$$

$$MO = \frac{2a}{11} \quad 5$$



$\rightarrow F = ma$

$$T_1' - T_2' = m\ddot{x} \quad 5$$

$$\frac{4\lambda}{3a}(a-x) - \frac{\lambda}{2a}(2a+x) = m\ddot{x} \quad 5$$

$$\frac{\lambda}{6a}(2a - 11x) = m\ddot{x}$$

$$\ddot{x} = \frac{-11\lambda}{6am}\left(x - \frac{2a}{11}\right) \quad 5$$

$X = x - \frac{2a}{11}$ இருபுறமும் t குறித்து வகையிட 5

$$\dot{X} = \dot{x}$$

$$\Rightarrow \ddot{X} = \ddot{x} \quad 5$$

$$\ddot{X} = \frac{-11\lambda}{6ma}X$$

$$\ddot{X} = -\omega^2 X \quad 5 \qquad \left(\omega^2 = \frac{11\lambda}{6ma}\right) 5$$

∴ துணிக்கை S·H·M ஐ ஆற்றும் 5

அலைவுகாலம் $T = \frac{2\pi}{\omega}$

$$= 2\pi\sqrt{\frac{6ma}{11\lambda}} \quad 5$$

அலைவுமையம் $X = 0$

$$x = \frac{2a}{11} \quad (\therefore \text{புள்ளி } O) \quad 5$$

துணிக்கை M இல் ஓய்விலிருந்து விடப்பட்டதால்,

S·H·M இன் வீச்சம் $c = MO$

$$= \frac{2a}{11} \quad 10$$

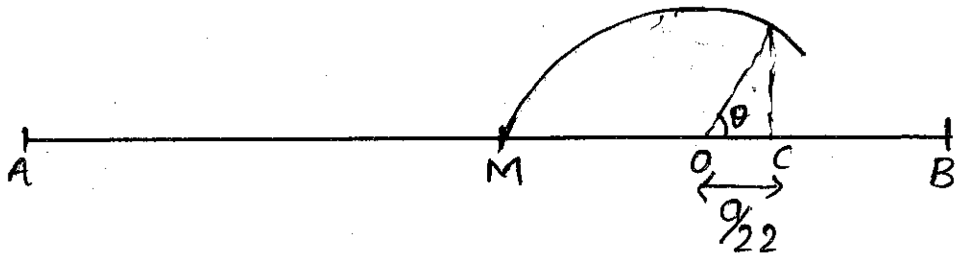
$$V^2 = \omega^2(C^2 - X^2) \quad 5$$

$x = \frac{5a}{22}$ ஆக $V = V_C$ என்க $\quad \therefore X = \frac{a}{22}$ 10

$$V_C^2 = \frac{11\lambda}{6ma}\left[\left(\frac{2a}{11}\right)^2 - \left(\frac{a}{22}\right)^2\right] \quad 5$$

$$= \frac{5\lambda a}{88m}$$

$$V_C = \frac{1}{2}\sqrt{\frac{5\lambda a}{22m}} \quad 5$$



10

$$\cos\theta = \frac{9/22}{29/11} \quad 10$$

$$= \frac{1}{4}$$

$$\theta = \cos^{-1}\left(\tfrac{1}{4}\right) \quad 10 \quad \left(0 < \theta < \pi/2\right)$$

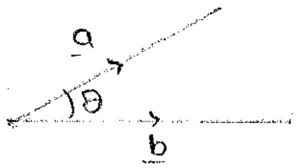
துணிக்கை $M \longrightarrow C$ இயங்க எடுத்த நேரம்

$$t_1 = \frac{\pi - \theta}{\omega} \quad 5$$

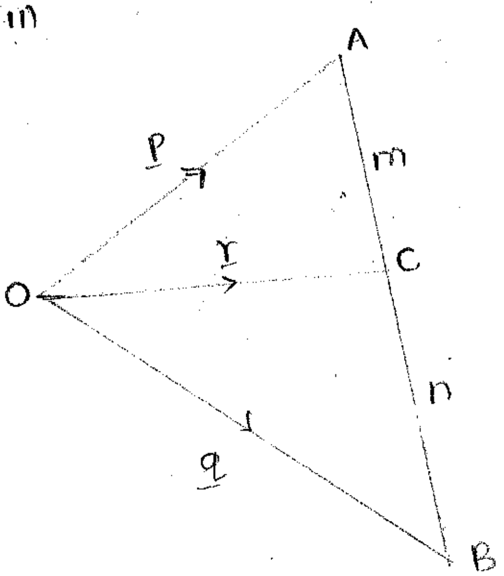
$$= \sqrt{\frac{6ma}{11\lambda}}\left[\pi - \cos^{-1}\left(\tfrac{1}{4}\right)\right] \quad 5$$

$\boxed{150}$

14) (a) i) $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$ 5



ii)



Δ ABC இல்

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\overrightarrow{AO} + \overrightarrow{OB}$

$= -\underline{p} + \underline{q}$ 5

$\overrightarrow{AB} = (\underline{q} - \underline{p})$ —— ①

$\dfrac{AC}{AB} = \dfrac{m}{(m+n)}$ 5

$AC = \left(\dfrac{m}{m+n}\right) AB$ —— ②

①, ② ⇒

$AC = \dfrac{m}{(m+n)}(\underline{q} - \underline{p})$ 5

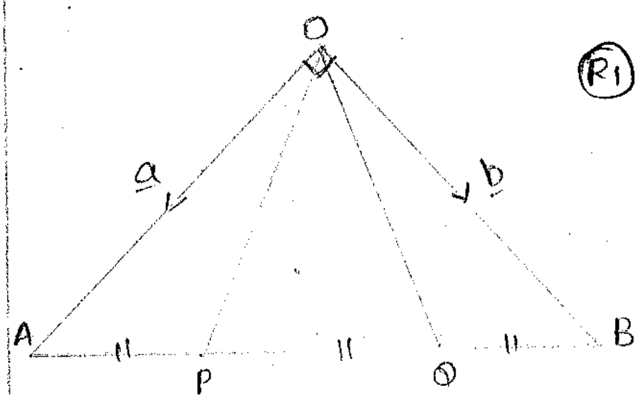
Δ OAC இல்

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ 5

$= \underline{p} + \left(\dfrac{m}{m+n}\right)(\underline{q} - \underline{p})$

$\overrightarrow{OC} = \left(\dfrac{m\underline{q} + n\underline{p}}{m+n}\right)$

$\underline{r} = \left(\dfrac{m\underline{q} + n\underline{p}}{m+n}\right)$ 5 —— (R_1)



(R_1) இல் $\underline{r} = OP$ $m \to 1$ $n \to 2$

$\underline{p} \to \underline{a}$

$\underline{q} \to \underline{b}$ ஆக

$\overrightarrow{OP} = \frac{2}{3}\underline{a} + \frac{1}{3}\underline{b}$ 5

(R_1) இல் $\underline{r} \longrightarrow OQ$ $m \to 2$ $n \to 1$

$\underline{p} \longrightarrow \underline{a}$

$\underline{q} \longrightarrow \underline{b}$ ஆக

$\overrightarrow{OQ} = \frac{1}{3}\underline{a} + \frac{2}{3}\underline{b}$ 5

$\vec{OP} \cdot \vec{OP} = \left(\frac{2}{3}\underline{a} + \frac{1}{3}\underline{b}\right)\left(\frac{2}{3}\underline{a} + \frac{1}{3}\underline{b}\right)$ (5)

$|\vec{OP}|^2 = \frac{4}{9}|\underline{a}|^2 + 4\,\underline{a}\cdot\underline{b} + \frac{1}{9}|\underline{b}|^2$

$(OP)^2 = \frac{4}{9}|\underline{a}|^2 + \frac{1}{9}|\underline{b}|^2$ (5) $\underline{\quad}$ (2) $\quad (\because \underline{a}\cdot\underline{b} = 0)$ (5)

இதேபோல $(OQ)^2 = \frac{4}{9}|\underline{b}|^2 + \frac{1}{9}|\underline{a}|^2$ ——— (3) (5)

(2) + (3) $\Rightarrow (OP)^2 + (OQ)^2 = \frac{5}{9}\left[|\underline{a}|^2 + |\underline{b}|^2\right]$

$|\underline{a}| = AO \qquad |\underline{b}| = OB$ (5)

$\frac{5}{9}\left[(AO)^2 + (OB)^2\right] = (OP)^2 + (OQ)^2$

$\frac{5}{9}(AB)^2 = (OP)^2 + (OQ)^2$ (5)

$\boxed{70}$

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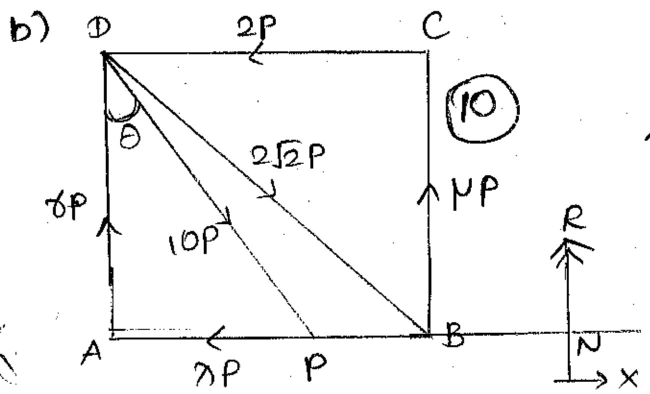
b)



$\tan\theta = \frac{3}{4}$

$\uparrow Y = \mu P + \gamma P - 10P\cos\theta$ (5) $- 2\sqrt{2}P\cos\pi/4$

$= (\mu + \gamma - 10)P$ (5)

$\overrightarrow{X} = -\lambda P - 2P + 10P\sin\theta + 2\sqrt{2}P\sin\pi/4$ (5)

$= (6 - \lambda)P$ (5)

i) தொகுதி சமநிலையிலிருப்பின்

$\uparrow Y = 0 \qquad\qquad \rightarrow X = 0$

$(\mu + \gamma - 10) = 0 \qquad (6 - \lambda) = 0$

$\mu + \gamma = 10$ (5) $\qquad \lambda = 6$ (5) $(R \neq 0)$

$D \circlearrowleft \quad \lambda P(4) - \mu P(4) = 0$

$\lambda = \mu$

$\therefore \mu = 6$ (5)

$\therefore \gamma = 4$ (5)

ii) $\gamma \neq 4$

$\lambda = \mu = 6$

$\uparrow Y = (\gamma - 4)P$ $\qquad Y \neq 0 \;,\; X = 0$ 5

$\rightarrow X = 0$ $\qquad \therefore$ தொகுதி தனிஇணைக்கு ஒடுங்கும்,

$D \circlearrowleft \quad \lambda P(4) - \mu P(4) = -Y(AN)$ 5

$0 = -Y(AN)$

$Y \neq 0 \Rightarrow \therefore AN = 0$ 5

$\therefore$ தாக்கக்கோடு AD இனூடு செல்கிறது.

iii) $\gamma = 2 \qquad \lambda = \mu = 6$

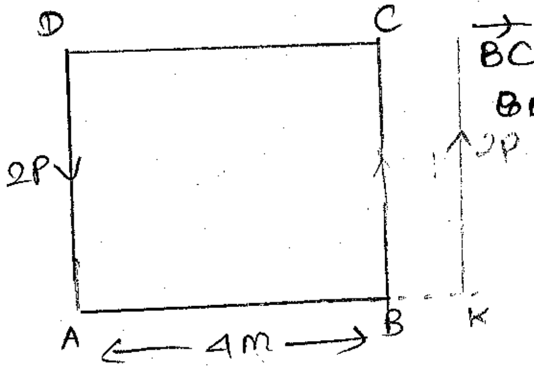
$\uparrow Y = (2-4)P = -2P$

$\rightarrow X = 0$ 5 $\quad \therefore$ தொகுதி தனிஇணைக்கு ஒடுங்கும்,

மேலதிக விசை சேர்க்கப்பட தொகுதி 8P Nm இணைக்கு ஒடுங்கும் எனின்

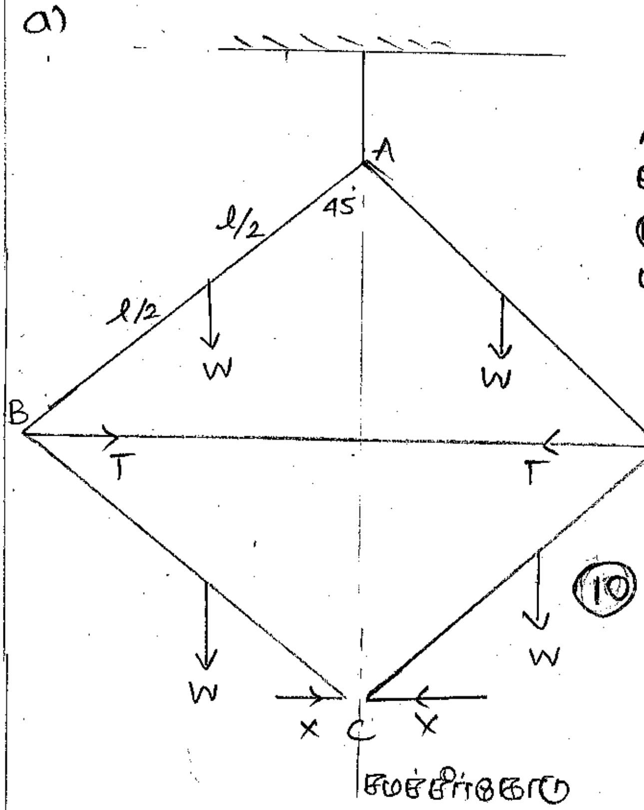
$A \circlearrowleft \quad 2P(AK) = 8P$

$AK = 4m$ 5 $\quad K \equiv B$.



$\overrightarrow{BC}$ இனூடு 2P விசை சேர்க்கப்பட வேண்டும். 5

15) a)



AC வழியே செல்லும் நிலைக்குத்துக் கோடு சமச்சீர்க்கோடு என்பதால் புள்ளி C யில் நிலைக்குத்து மறுதாக்கம் பூச்சியம். 5

BC க்கு

B ↺ $\Rightarrow$

$$w\left(\frac{l}{2}\right)\cos 45 = x(l)\sin 45 \quad 10$$

$$\frac{W}{2} = x \tan 45$$

$$x = \frac{W}{2} \quad 5$$

(AB + BC) க்கு A ↻

$$w\left(\frac{l}{2}\right)\cos 45° + w\left(\frac{l}{2}\right)\cos 45° + x(2l)\sin 45 + T(l)\sin 45 = 0 \quad 10$$

$$2W + T = 0$$

$$T = -2W \quad 5$$

கயிற்றின் நீளம் L என்க.

ஊக்கின் விதிக்கு $\Rightarrow$ $T = \left(\frac{\lambda}{L}\right) e$ 5

$$2W = \frac{3W}{L}\left(2l\sin 45° - L\right) \quad 5$$

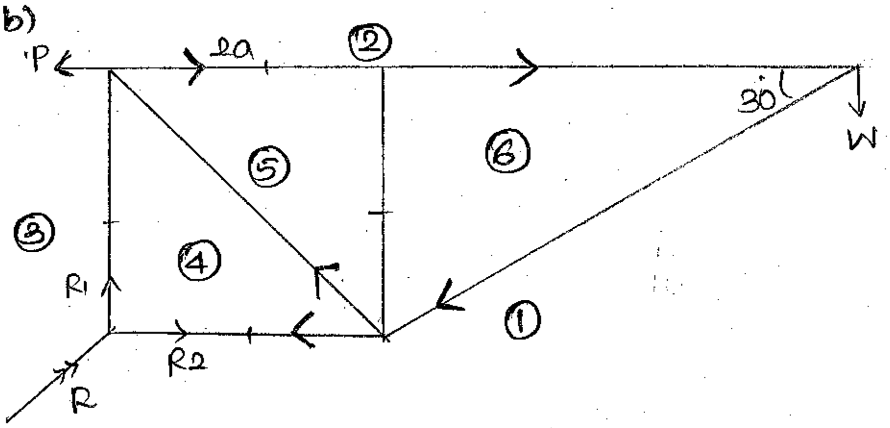
$$2L = 3\left(\sqrt{2}\,l - L\right)$$

$$5L = 3\sqrt{2}\,l$$

$$L = \frac{3\sqrt{2}}{5}\,l \quad 5$$

60

b)



i) $A \circlearrowleft \quad W(2a + 4a\cos 30) - P(2a) = 0$

$$P = (1+\sqrt{3})W \quad ⑤$$

ii) $\uparrow \quad R_1 - W = 0$

$$R_1 = W \quad ⑤$$

$\rightarrow$

$$R_2 - P = 0$$

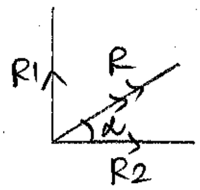
$$R_2 = (1+\sqrt{3})W \quad ⑤$$

A இல் மறுதாக்கம்

$$R = \sqrt{R_1^2 + R_2^2}$$

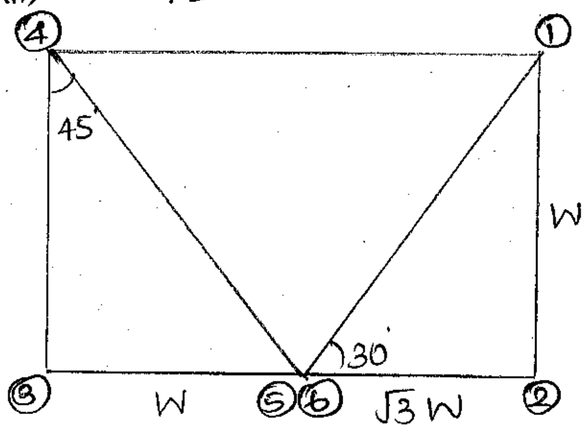
$$= \sqrt{W^2 + (1+\sqrt{3})^2 W^2}$$

$$= \sqrt{5+2\sqrt{3}}\, W \quad ⑤$$



$$\tan\alpha = \frac{R_1}{R_2} = \frac{1}{(1+\sqrt{3})} \quad ⑤$$

iii)



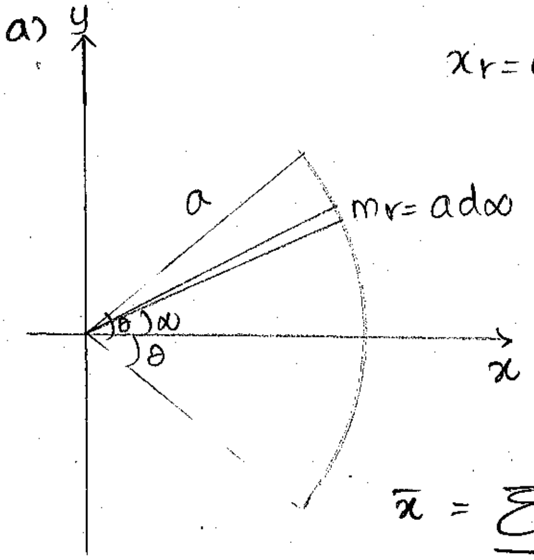
கோல்	உதைப்பு	இழுவை
AB	W	–
BC	–	W
CE	–	–
AE	$(1+\sqrt{3})W$	–
BE	–	$\sqrt{2}W$
CD	–	$\sqrt{3}W$
DE	2W	–

⑩ + ⑩ + ⑩

⑭ + ㉑

90

16) a)



$x_r = a\cos\alpha \quad (d\alpha \to 0)$

ஈர்வைமையம் $G \equiv (\bar{x}, \bar{y})$ என்க.

நீள அடர்த்தி ρ என்க.

x - அச்சு சமச்சீரச்சு. $\therefore$ ஈர்வை மையம் x அச்சிலிருக்கும்.

$\bar{y} = 0$ 5

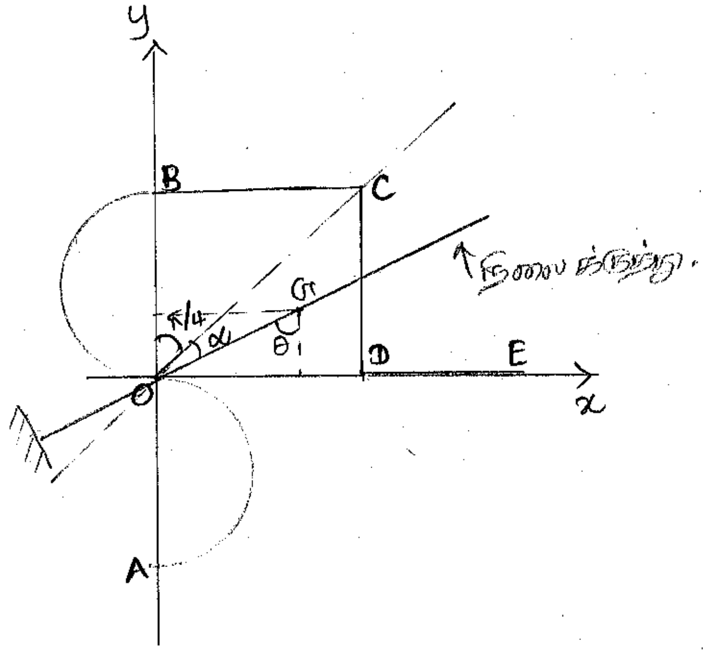
$$\bar{x} = \frac{\sum m_r x_r}{\sum m_r} = \frac{\int_{-\theta}^{\theta} a\,d\alpha\,\rho\,a\cos\alpha}{\int_{-\theta}^{\theta} a\,d\alpha\,\rho} \quad 5$$

$$= \frac{a^2\rho\int_{-\theta}^{\theta}(\cos\alpha)\,d\alpha}{a\rho\int_{-\theta}^{\theta}(1)\,d\alpha}$$

$$= \frac{a\,[-\sin\alpha]_{-\theta}^{\theta}}{[\alpha]_{-\theta}^{\theta}} \quad 5$$

$$= \frac{a\sin\theta}{\theta} \quad 5$$

$\boxed{20}$



உருவம்	திணிவு	ஈர்வை மையம் x அச்சிலிருந்து	ஈர்வை மையம் y அச்சிலிருந்து	
வில் OA	$\pi \frac{l}{2} \rho$	$-\frac{l}{2}$	$\frac{l}{\pi}$	10
வில் OB	$\pi \frac{l}{2} \rho$	$\frac{l}{2}$	$-\frac{l}{\pi}$	10
கோல் BC	$l\rho$	l	$\frac{l}{2}$	10
கோல் CD	$l\rho$	$\frac{l}{2}$	l	10
கோல் DE	$l\rho$	0	$\frac{3l}{2}$	10
கூட்டு உடல்	$(\pi+3)l\rho$	$\bar{y}$	$\bar{x}$	10

x அச்சு ↻

$$(3+\pi)\, l\rho\, \bar{y} = \frac{\pi l \rho}{2}\left(-\frac{l}{2}\right) + \frac{\pi l\rho}{2}\left(\frac{l}{2}\right) + l\rho\left(\frac{l}{2}\right) + l\rho(0)$$

5 5

$$\bar{y} = \frac{3l}{2(\pi+3)}$$

$$= \frac{l}{2\left(\frac{\pi}{3}+1\right)} = \frac{l}{2k}$$ 5

y அச்சு ↻

$$(3+\pi)\, l\rho\, \bar{x} = \frac{\pi l\rho}{2}\left(\frac{l}{\pi}\right) + \frac{\pi l \rho}{2}\left(\frac{l}{\pi}\right) + l\rho(l) + \rho l\left(\frac{l}{2}\right) + \rho l\left(\frac{3l}{2}\right)$$

5 5

$$\bar{x} = \frac{3l}{(\pi+3)}$$

$$= \frac{l}{\left(\frac{\pi}{3}+1\right)} = \frac{l}{k}$$ 5

$$\alpha = \theta - \frac{\pi}{4}$$

$\boxed{35}$

O இலிருந்து கட்டித் தொங்கவிடப்பட்ட பின் ;
OC நிலைக்குத்துடன் ஆக்கும் கோணம் α எனக.

$$\tan\theta = \frac{\bar{x}}{\bar{y}} = \frac{l/k}{l/2k} = 2 \quad (5)$$

$$\theta = \tan^{-1}(2)$$

$$\sin\alpha = \sin(\theta - \pi/4)$$

$$= \sin\theta\cos\pi/4 - \cos\theta\sin\pi/4$$

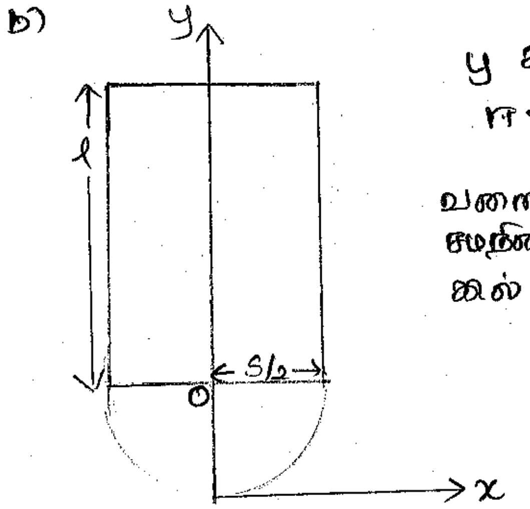
$$= 2/\sqrt{5} \cdot 1/\sqrt{2} - 1/\sqrt{5} \cdot 1/\sqrt{2}$$

$$= 1/\sqrt{10}$$

$$\alpha = \sin^{-1}(1/\sqrt{10}) \quad (5)$$

$\boxed{10}$

b)



y அச்சு சமச்சீரச்சு. ∴ பொருளின் ஈர்வை மையம் y அச்சிலிருக்கும்,

வளைபரப்பைத் தொட்டுக் கொண்டு சமநிலை அமைவதால் ஈர்வை மையம் O இல் இருக்கும்.

$$G \equiv (0, s/2) \quad (5)$$

கனவளவு அடர்த்தி ρ என்க.

உருவம்	திணிவு	ஈர்வைமையம் x அச்சிலிருந்து	
½ கோளம்	$\frac{2}{3}\pi(s/2)^3\rho$	$5s/16$	(5)
உருளை	$\pi(s/2)^2 l\rho$	$s/2 + l/2$	(5)
கூட்டு உடல்	$\frac{\pi s^2\rho}{12}(3l+s)$	$s/2$	(5)

x அச்சு $\circlearrowleft$

$$\frac{\pi s^2\rho}{12}(3l+s) \cdot s/2 = \frac{\pi s^3\rho}{12}(5s/16) + \frac{\pi s^2 l\rho}{4}\frac{(s+l)}{2} \quad (5) \quad (5)$$

$$(s/l)^2 = 8$$

$$s/l = 2\sqrt{2} \quad (5)$$

$\boxed{35}$

17) a) $A \Rightarrow \left.\begin{matrix} W - 1 \\ B - 2 \\ R - 3 \end{matrix}\right\} 6$ $\quad B \Rightarrow \left.\begin{matrix} W - 2 \\ B - 1 \\ R - 1 \end{matrix}\right\} 4$

$C \Rightarrow \left.\begin{matrix} W - 4 \\ B - 5 \\ R - 3 \end{matrix}\right\} 12$

$$P(W) = P(W/A) \cdot P(A) + P(W/B) \cdot P(B) + P(W/C) \cdot P(C)$$

$$= \frac{1}{6} \times \frac{1}{3} + \frac{2}{4} \times \frac{1}{3} + \frac{4}{12} \times \frac{1}{3}$$

10 10 10

$$= \frac{1}{3}$$ 5

Bayes Theorm

$$P(B/W) = \frac{P(W/B)\, P(B)}{P(W)}$$ 10

$$= \frac{\frac{2}{4} \times \frac{1}{3}}{\frac{1}{3}}$$ 5 5 5

$$= \frac{1}{2}$$ 5

$\boxed{65}$

வகுப்பாயிடை	மீடிறன் (f)	ந.பெ (y)	விலகல் (d)	fd	fd^2
5.6 – 5.8	2	5.7	–3	–6	18
5.8 – 6.0	7	5.9	–2	–14	28
6.0 – 6.2	16	6.1	–1	–16	16
6.2 – 6.4	21	6.3	0	0	0
6.4 – 6.6	12	6.5	1	12	12
6.6 – 6.8	2	6.7	2	4	8
				$\Sigma fd = (-20)$	$\Sigma fd^2 = 82$
				10	10

Alternate method :-

வகுப்பாயிடை	மீடிறன் (f)	ந.பெ (y)	fy	fy^2
	5	5	5	
5.6 – 5.8	2	5.7	11.4	64.98
5.8 – 6.0	7	5.9	41.3	243.67
6.0 – 6.2	16	6.1	97.6	595.36
6.2 – 6.4	21	6.3	132.3	833.49
6.4 – 6.6	12	6.5	78	507
6.6 – 6.8	2	6.7	13.4	89.78
			$\Sigma fy = 374$	$\Sigma fy^2 = 2334.28$
			5	5

(i) $\bar{x} = A + i\dfrac{\Sigma fd}{n}$ 5

$= 6.3 + 0.2 \times \left(-\dfrac{20}{60}\right)$

$= 6.23$ 5

(ii) $Sx = i\sqrt{\dfrac{\Sigma fd^2}{n} - \left(\dfrac{\Sigma fd}{n}\right)^2}$ 20

$= 0.2\sqrt{82/60 - 1/9}$

$= 0.224$ 20

Alternate method :-

$\bar{x} = \dfrac{\Sigma fy}{\Sigma f}$ 10

$= \dfrac{374}{60}$ 5

$= 6.23$ 5

$Sx = \sqrt{\dfrac{\Sigma fy^2}{\Sigma f} - \bar{x}^2}$ 10

$= \sqrt{\dfrac{2334.28}{60} - (6.23)^2}$ 5

$= 0.224$ 5

(iii) $M_O = L_O + \frac{i\,[f_1 - f_2]}{[f_1 - f_2] + [f_1 - f_2]}$ 10

$= 6.2 + \frac{0.2 \times 5}{(5+9)}$ 5

$= 6.27$ 5

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