பகுதி A

1) $n = 1$ ஆக

$L.H.S = 4C1 + 1 = 5$

$R.H.S = 1\ C_{2\times1+3} = 5$ $\quad L.H.S. = R.H.S$

$\therefore n = 1$ க்கு முடிவு உண்மையாகும்

$n = p$ க்கு முடிவு உண்மை என்க $(p \in Z^+)$

$$\sum_{r=1}^{p} (4r+1) = p(2p+3)$$

இருபுறமும் $(4p+5)$ ஐ கூட்ட

$$\sum_{r=1}^{p} (4r+1) + 4p+5 = 2p^2+3p+4p+5$$

$$\sum_{r=1}^{p+1} (4r+1) = 2p^2+7p+5$$

$$= (p+1)(2p+5)$$

$$= (p+1)[2(p+1)+5]$$

$\therefore n = p+1$ க்கு முடிவு உண்மையாகும்.

$n = 1$ க்கு முடிவு உண்மை. $n = p$ க்கு முடிவு உண்மை எனின் $n = p+1$ க்கும் முடிவு உண்மையாகும். ஆகவே எல்லா $n \in Z^+$ க்கும் முடிவு உண்மையாகும்.

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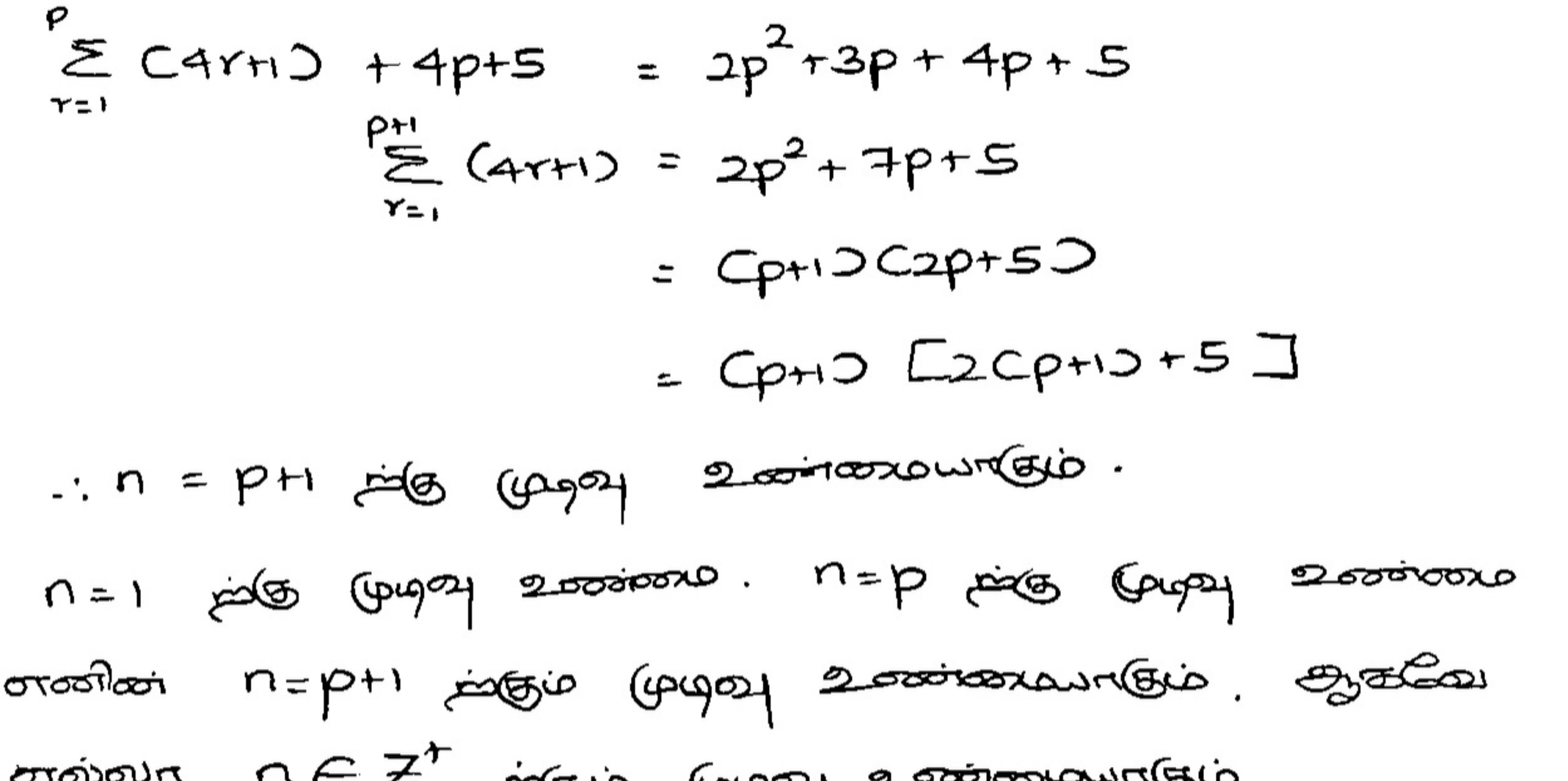
2)



$\frac{t}{2} = x$ என்க

$-2x = 2-x$

$x = -2$

$|x-2| < 2(1+|2x+1|)$

$x < -2$ or $x > 0$

$|\frac{t}{2}-2| < 1+|2\frac{t}{2}+1|$

$|t-4| < 2(1+|t+1|)$

$t < -2$ or $t > 0$

$\frac{x}{2} < -2$ or $\frac{x}{2} > 0$

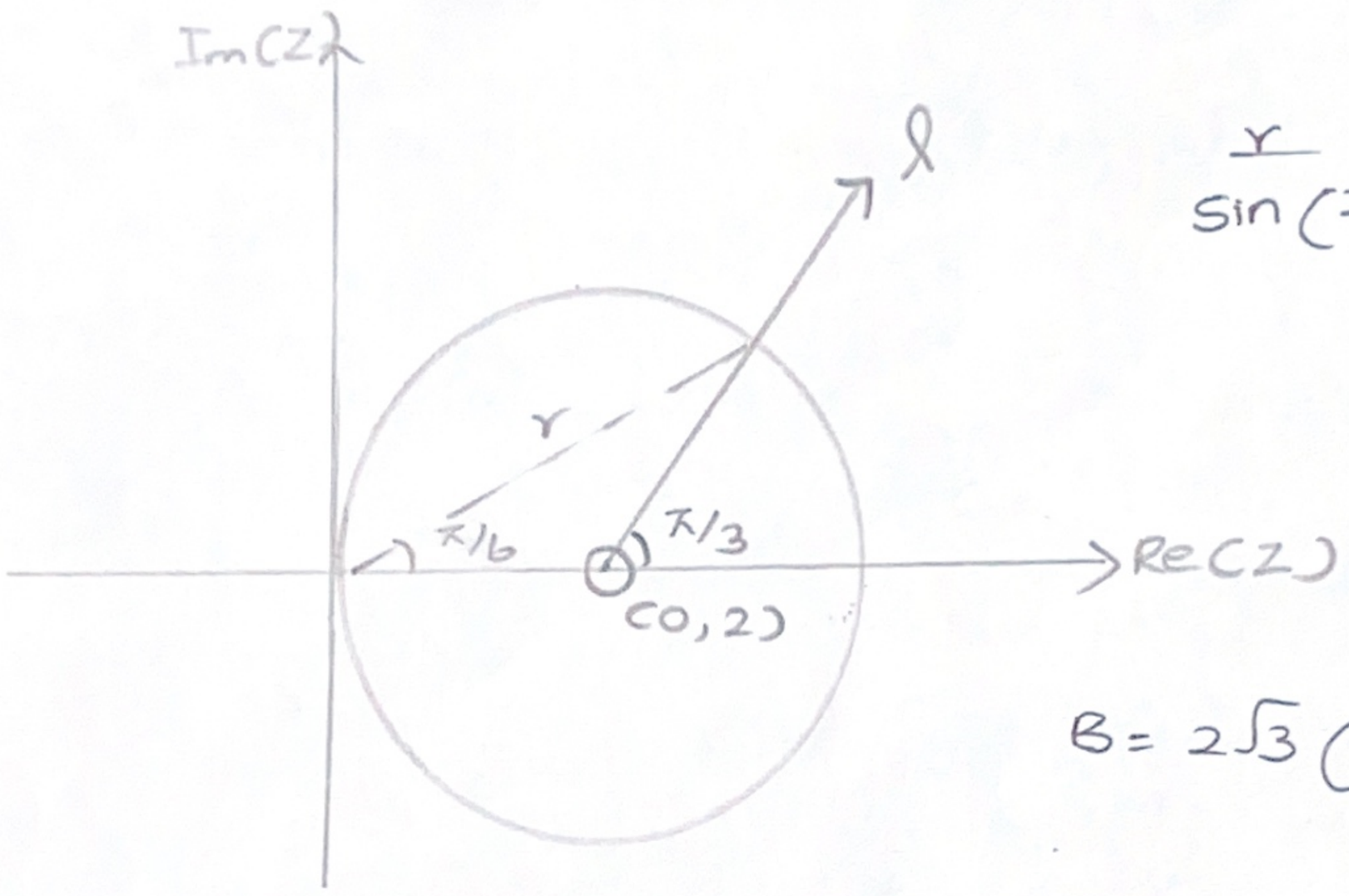
$x < -4$ or $x > 0$

3) $\text{Arg}(z-2i) = \pi/3$

$\text{Arg}[z-(0+2i)] = \pi/3$

$|z-2i| = 2$

$|z-(0+2i)| = 2$



$$\frac{r}{\sin(2\pi/3)} = \frac{2}{\sin \pi/6}$$

$$\frac{r}{\sqrt{3}} = 2$$

$$r = 2\sqrt{3}$$

$$B = 2\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\pi/6\right)$$

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4) $\left(x^2 + \frac{3k}{x}\right)^8 = (x^2)^8 + \dots {}^8C_r (x^2)^{8-r} \left(\frac{3k}{x}\right)^r + \dots \left(\frac{3k}{x}\right)^8$

$x^1 = \frac{x^{16-2r}}{x^r}$

$x^1 = x^{16-3r}$

$16 - 3r = 1$

$r = 5$

$x^{16-3r_1} = x^4$

$16 - 3r_1 = 4$

$r = 4$

${}^8C_5 (3k)^5 = {}^8C_4 (3k)^4$

$\frac{8!}{3!\,5!}(3k) = \frac{8!}{4!\,4!}$

$k = \frac{5}{4\times 3}$

$= \frac{5}{12}$

5) $\lim_{x \to \pi/2} \frac{(2x-\pi)\cos x}{2\cos^2 x - (\frac{\pi}{2}-x)^2 \sin x}$

$= \lim_{x \to \pi/2} \frac{-2(\pi/2 - x)\sin(\frac{\pi}{2}-x)}{2\sin^2(\frac{\pi}{2}-x) - (\frac{\pi}{2}-x)^2 \sin x}$

$= -2 \lim_{x \to \pi/2} \frac{\sin(\pi/2-x)/(\pi/2-x)}{2\left[\frac{\sin(\pi/2-x)}{(\pi/2-x)}\right]^2 - \sin x}$

$= -2 \lim_{(\frac{\pi}{2}-x)\to 0} \frac{\sin(\pi/2-x)/(\pi/2-x)}{\lim_{(\frac{\pi}{2}-x)\to 0} 2\left[\frac{\sin(\pi/2-x)}{(\pi/2-x)}\right]^2 - \lim_{x\to 0} \sin x}$

$= \frac{-2 \times 1}{2-1} = -2$

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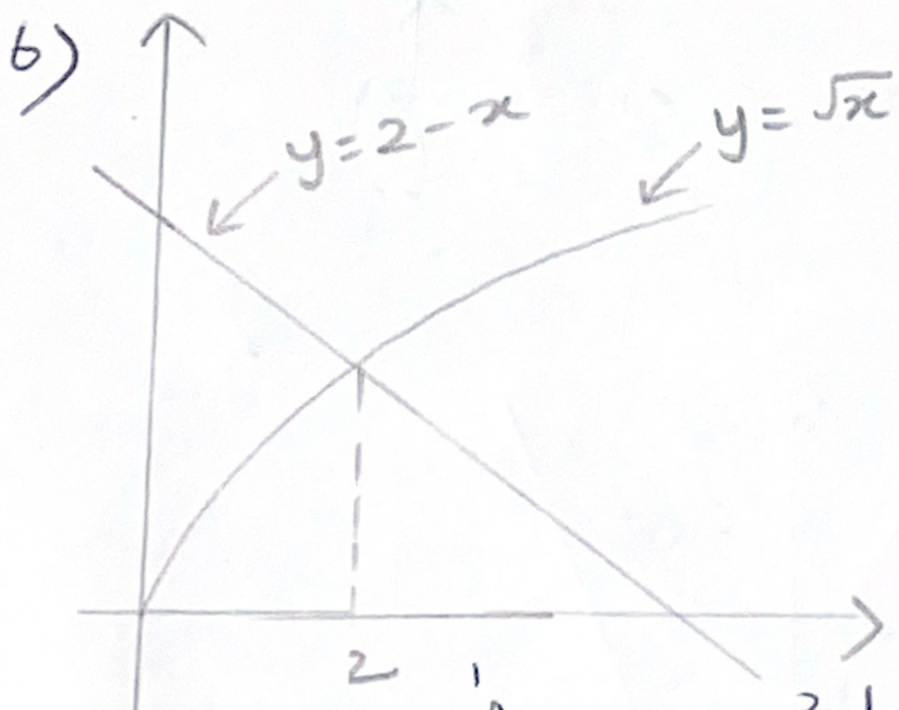
6)



$y = \sqrt{x}$ — ① $\quad y = 2-x$ — ②

$2-x = \sqrt{x}$

$x = (2-x)^2$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1$ or $x = 4$

தேவையான $= \pi \int_0^1 (2-x)^2 dx - \pi \int_0^1 (\sqrt{x})^2 dx$

$= \pi \int_0^1 4 - 4x + x^2 dx - \pi \int_0^1 x\, dx$

$= \pi \left(4x - \frac{4x^2}{2} + \frac{x^3}{3}\right)_0^1 - \pi\left(\frac{x^2}{2}\right)_0^1$

$= \pi\left(4 - 2 + \frac{1}{3} - \frac{1}{2}\right)$

$= \frac{11}{6}\pi$ கன அலகு

7) $x = \frac{a}{2}(t + 1/t)$ $\quad y = a/2\,(t - \frac{1}{t})$

$\frac{dx}{dt} = \frac{a}{2}(1 - \frac{1}{t^2})$ $\quad \frac{dy}{db} = \frac{a}{2}(1 + \frac{1}{t^2})$

$t = 2 \Rightarrow x = a/2\,(2 + \frac{1}{2})$ $\quad t = 2 \Rightarrow y = 3a/4$

$= 5a/4$

$\frac{dy}{dx} = \frac{dy}{db} \times \frac{db}{dx} = \frac{a/2\,(1 + \frac{1}{t^2})}{a/2\,(1 + \frac{1}{t^2})} = \frac{(1 + t^2)}{(t^2 - 1)}$

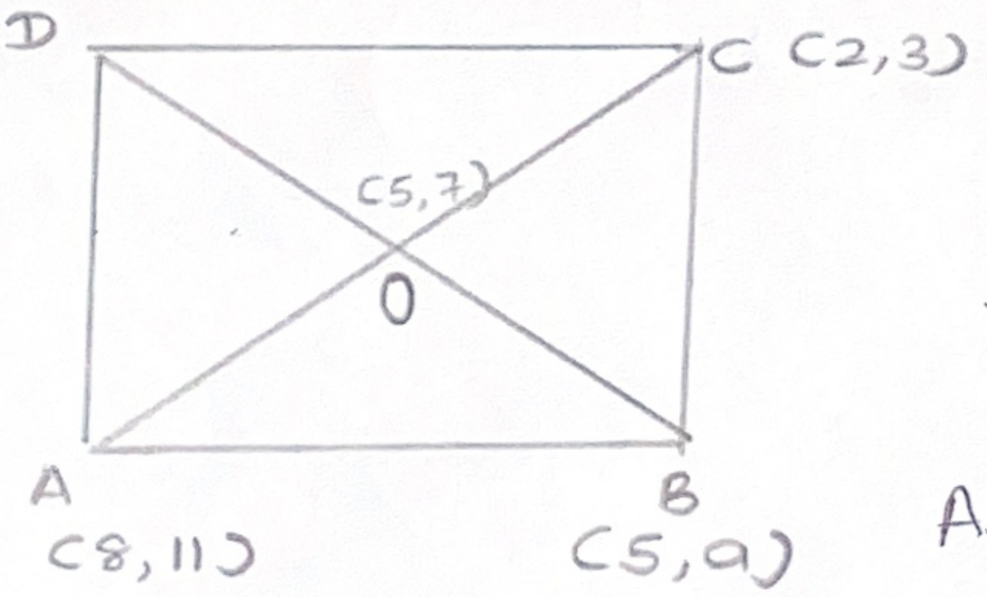
$\left(\frac{dy}{dx}\right)_{t=2} = 5/3$ (தொடலியின் படித்திறன் = $10/3$

தொடலியின் சமன்பாடு =>

$\frac{y - 3a/2}{x - 5a/4} = \frac{5}{3}$ $\quad 3y - 9a/2 = 5x - 5a/4$

$10x - 6y - 8a = 0$

8)



$O = [5, 7]$

$B = (5, a)$

$AB \perp BC$

$\frac{(a-11)}{(5-8)} \times \frac{(a-3)}{(5-2)} = (-1)$

ஏனைய உச்சிகளின் ஆள்கூறு

(5,12) (5,2)

$\frac{a^2 - 14a + 33}{-9} = -1$

$a^2 - 14a + 33 = 9$

$a^2 - 14a + 24 = 0$

$(a - 12)(a - 2) = 0$

9) $S_1 = x^2 + y^2 + 2gx + 2fy + c = 0$

S_1 (3, 0) ஐ திருப்தியாக்கும்

$9 + 6g + c = 0 \quad -①$

$r^2 = g^2 + f^2 - c$

$g^2 = c \qquad (r = -f)$

$① \Rightarrow 9 + 6g + g^2 = 0$

$(g+3)^2 = 0$

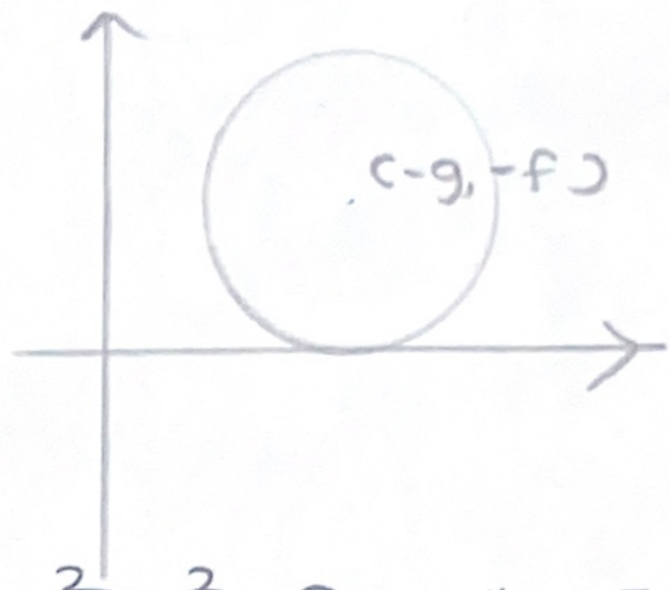
$g = (-3)$

$c = 9$

மையம் = $(3, -5)$

ஆரை $\Rightarrow r = \sqrt{(-3)^2 + 5^2 - 9}$

$= 5$ அலகு



$S_2 = x^2 + y^2 - 8x - 4y - 5 = 0$

S_1, S_2 ஐ செங்கோணத்தில்

வெட்டுவதால்

$2(-3)(-4) + 2(f)(-2)$

$= 9 + (-5)$

$24 - 4f = 4$

$4f = 20$

$f = 5$

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10) $\text{cosec}^2\theta - \cot^2\theta = 1$

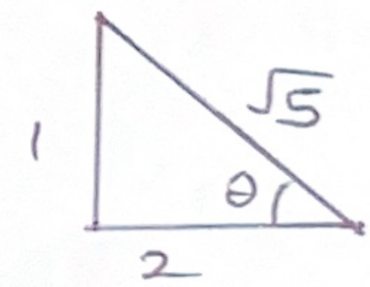
$\text{cosec}^2\theta = 1 + 4 = 5$

$\text{cosec}\theta = -\sqrt{5} \qquad \left(\frac{3\pi}{2} < \theta < 2\pi\right)$

$\sin\theta = \frac{-1}{\sqrt{5}}$

$\sec\theta - \text{cosec}\theta = \frac{\sqrt{5}}{2} - (-\sqrt{5})$

$= \frac{3\sqrt{5}}{2}$



11)

a) $x^2 - bx - c = 0$

$\alpha + \beta = b \qquad \alpha\beta = -c$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta \qquad = b^2 + 2c$

$x^2 - px - q = 0$

$\gamma + \delta = p \qquad \gamma\delta = -q$

$L.H.S = (\alpha+\gamma)(\alpha+\delta)(\beta+\gamma)(\beta+\delta)$

$= [\alpha^2 + (\gamma+\delta)\alpha + \gamma\delta)][\beta^2 + (\gamma+\delta)\beta + \gamma\delta]$

$= (\alpha^2 + p\alpha - q)(\beta^2 + p\beta - q)$

$= (\alpha\beta)^2 + p\alpha^2\beta - \alpha^2 q + p\alpha\beta^2 + p^2\alpha\beta - p\alpha q - q\beta^2 - pq\beta + q^2$

$= c^2 + q^2 - (\alpha^2+\beta^2)q + p\alpha\beta(\alpha+\beta) - p^2c - pq(\alpha+\beta)$

$= c^2 + q^2 - (b^2+2c)q - pbc - p^2c - pqb$

$= (c^2 - 2cq + q^2) - [b^2q + pqb + pbc + p^2c]$

$= (c-q)^2 - [bq(b+p) + pc(b+p)]$

$= (c-q)^2 - (bq+pc)(b+p)$ —— (R)

$(c-q)^2 = (b+p)(bq+cp) \Leftrightarrow (c-q)^2 - (b+p)(bq+cp) = 0$

$\Leftrightarrow (\alpha+\gamma)(\alpha+\delta)(\beta+\gamma)(\beta+\delta) = 0$

$\Leftrightarrow (\alpha+\gamma) = 0$ or $(\alpha+\delta) = 0$ or $(\beta+\gamma) = 0$ or $(\beta+\delta) = 0$

$\Leftrightarrow \alpha = -\gamma$ or $\alpha = -\delta$ or $\beta = -\gamma$ or $\beta = -\delta$

$\Leftrightarrow x^2 - bx - c = 0$ எனும் இருபடிச்சமண்பாட்டின்

ஒரு மூலமானது $x^2 - px - q = 0$ ன் ஒரு மூலத்தின் மறை பெறுமானத்திற்கு சமன்

(R) இவ் இருந்து உய்த்தறிதல்

$b \Rightarrow 3$, $c \Rightarrow 10k$, $p \Rightarrow k$, $q \Rightarrow 2k$

$(10k-2k)^2 = (3k+k)(10k^2+6k)$

$k(k-1)(5k-9) = 0$

$k = 1$ or $k = 9/5$ or $k = 0$

b)

$f(x) = x^4 + x^3 - (2+d)x^2 + d(c+1)x - cd$

$f(c) = 0$

$c^4 + c^3 - (2+d)c^2 + d(c+1)c - cd = 0$

$c^4 + c^3 - (2+d)c^2 + dc^2 + dc - cd = 0$

$c^4 + c^3 - 2c^2 = 0$

$(c+2)(c-1) = 0$

$c = 1$ or $c = -2$ ஆனால் $c \in R^+$ என்பதால்

$c = 1$ ஏற்கும்

$f(x) = (x-d)\phi(x) + cd(d-1)$

$x^4 + x^3 - (2+d)x^2 + d(c+1)x - cd = (x-d)\phi(x) + cd(d-1)$

$c = 1$ என்பதால்

$x^4 + x^3 - (2+d)x^2 + 2dx - d = (x-d)\phi(x) + d(d-1)$

$x = d \Rightarrow$

$d^4 + d^3 - (2+d)d^2 + 2d^2 - d = d(d-1)$

$d^4 - d^2 = 0$

$d^2(d^2-1) = 0$

$d = 0$ or $d = 1$ or $d = -1$ but $d \in R^+$ ஆதலினால்

$d = 1$

$f(x) = x^4 + x^3 - (2+d)x^2 + d(c+1)x - cd$

$f(x) = x^4 + x^3 - 3x^2 + 2x - 1$

$f(x) = (x+c+d)^3 g(x) + Ax + B$

$= (x+2)^3 g(x) + Ax + B$

$f'(x) = g(x)\frac{d}{dx}(x+2)^3 + (x+2)^3\frac{d}{dx}g(x) + A$

$f'(x) = 3(x+2)^2 g(x) + (x+2)^3 g'(x) + A$

$4x^3 + 3x^2 - 6x + 2 = 3(x+2)^2 g(x) + (x+2)^3 g'(x) + A$

$x = -2$ என்னிட

$4(-8) + 3(4) - 6(-2) + 2 = A \qquad A = -6$

① $\Rightarrow$ $x = -2$ என்னிட

$x^4 + x^3 - 3x^2 + 2x - 1 = (x+2)^2 g(x) + Ax + B$

$16 - 8 - 12 \quad -12 - 1 = 12 + B \qquad B = -29$

12)

a) $U_1 + U_2 + \cdots + U_{r-1} + U_r = \frac{r}{12}(r+1)(r+2)(r+3)$ — ①

$U_1 + U_2 + \cdots + U_{r-1} = \frac{(r-1)r(r+1)(r+2)}{12}$ — ②

① - ② ⇒

$$U_r = \frac{r(r+1)(r+2)}{3}$$

$$\frac{1}{U_r} = \frac{3}{r(r+1)(r+2)} = \frac{1}{2} \times \frac{3}{(r+1)}\left[\frac{1}{r} - \frac{1}{r+2}\right]$$

$$= \frac{3}{2}\left[\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right]$$

$\therefore k = \frac{3}{2}$ $\quad f(r) = \frac{1}{r(r+1)}$

$r=1 \Rightarrow \frac{1}{kU_1} = f(1) - f(2)$

$r=2 \Rightarrow \frac{1}{kU_2} = f(2) - f(3)$

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$r=n \Rightarrow \frac{1}{kU_n} = f(n) - f(n+1)$

$$\frac{1}{k}\sum_{r=1}^{n}\frac{1}{U_r} = f(1) - f(n+1)$$

$$\sum_{r=1}^{n}\frac{1}{U_r} = \frac{3}{2}\left[\frac{1}{1(2)} - \frac{1}{(n+1)(n+2)}\right]$$

$$= \frac{3}{2}\left[\frac{n^2+3n+2-2}{2(n^2+3n+2)}\right]$$

$$= \frac{3}{4}\left[\frac{n^2+3n}{(n+1)(n+2)}\right]$$

$S_n = \frac{3}{4}\left[\frac{n^2+3n}{(n+1)(n+2)}\right]$ $\qquad S_n = \frac{3 + 9/n}{4(1+1/n)(1+2/n)}$

$n \to \infty$ ஆக $S_n \to \frac{3}{4}$ ஆகும். $(n \to \infty$ ஆக $1/n \to 0)$

∴ தொடர் ஒருங்கும்

$$\sum_{r=1}^{n} \frac{1}{u_r} = \frac{3(1)(4)}{4(2)(3)} = \frac{1}{2}$$

$$\sum_{r=1}^{n=1} \leq \sum_{r=1}^{n} \frac{1}{u_r} < \sum_{r=1}^{n\to\infty} \frac{1}{u_r}$$

$$\frac{1}{2} \leq \frac{3n(n+3)}{4(n+1)(n+2)} < \frac{3}{4}$$

$$2 \leq \frac{3n(n+3)}{(n+1)(n+2)} < 3$$

$$2 \leq 3\left[1 - \frac{2}{(n+1)(n+2)}\right] < 3$$

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i) குழுவில் A யும் B யும் இருத்தல் = ${}^{10}C_2 = \frac{10!}{8!2!} = 45$

ii) வகை 1

B மட்டும் இருக்குமாறு 4 பேரை தெரிதல் = ${}^{10}C_3$

$= \frac{10!}{7!3!} = 120$

வகை 2

A மட்டும் இருக்குமாறு 4 பேரை தெரிதல் = ${}^{10}C_3 = 120$

∴ குழுவில் A / B இருத்தல் = 240

iii) A, B இருவரும் இடம்பெறாமை = ${}^{10}C_4$

$= \frac{10!}{6!4!}$

$= 210$

13)

a) $A = \begin{pmatrix} 0 & -2 & 1 \\ 2 & 1 & 0 \end{pmatrix}_{2\times 3}$ $\quad B = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}_{3\times 2}$

$$AB = \begin{pmatrix} 0 & -2 & 1 \\ 2 & 1 & 0 \end{pmatrix}_{2\times 3}\begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}_{3\times 2} = \begin{pmatrix} -1 & -2 \\ 1 & 3 \end{pmatrix}_{2\times 2}$$

$$C = \begin{pmatrix} -1 & -2 \\ 1 & 3 \end{pmatrix}_{2\times 2} \quad \text{—①}$$

$$C^{-1} = \frac{1}{(-2+3)}\begin{pmatrix} -3 & -2 \\ 1 & 1 \end{pmatrix}_{2\times 2} \qquad C^{-1} = \begin{pmatrix} -3 & -2 \\ 1 & 1 \end{pmatrix}$$

$$CDC^{-1} = C2C^2 - 3C)$$

$$CDCC^{-1}C) = 2C^3 - 3C^2$$

$$CD = C\,C2C^2 - 3C)$$

$$D = 2C - 3C \quad \text{—①}$$

$$C^2 = \begin{pmatrix} -1 & -2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -1 & -2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ 2 & 7 \end{pmatrix}_{2\times 2}$$

①⇒

$$D = \begin{pmatrix} -2 & -8 \\ 4 & 14 \end{pmatrix} - \begin{pmatrix} -3 & -6 \\ 3 & 9 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 1 & 5 \end{pmatrix}_{2\times 2}$$

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b) $Z_1 = r_1\,C\cos\theta_1 + i\sin\theta_1)$ $\qquad |Z_1| = r_1$

$Z_2 = r_2\,C\cos\theta_2 + i\sin\theta_2)$ $\qquad |Z_2| = r_2$

$$Z_1 - Z_2 = r_1\,C\cos\theta_1 + i\sin\theta_1) - r_2\,C\cos\theta_2 + i\sin\theta_2)$$

$$|Z_1 - Z_2| = CZ_1 - Z_2)\,C\overline{Z_1 - Z_2})$$

$$= [(r_1\cos\theta_1 - r_2\cos\theta_2) + i(r_1\sin\theta_1 - r_2\sin\theta_2)]$$

$$[(r_1\cos\theta_1 - r_2\cos\theta_2) - i(r_1\sin\theta_1 - r_2\sin\theta_2)]$$

$= (r_1\cos\theta_1 - r_2\cos\theta_2)^2 - i^2(r_1\sin\theta_1 - r_2\sin\theta_2)^2$

$= (r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1 - r_2\sin\theta_2)^2$

$= r_1^2(\cos^2\theta_1 + \sin^2\theta_1) + r_2^2(\cos^2\theta_2 + \sin^2\theta_2) - 2r_1r_2$

$(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$

$= r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)$

$= |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$z_1\bar{z}_2 = r_1r_2[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

$|1 - z_1\bar{z}_2| = 1 + (r_1r_2)^2 - 2(r_1r_2)\cos(\theta_1 - \theta_2)$ — ①

$|z_1 - z_2|^2 = r_1^2 + r_2^2 - 2(r_1r_2)\cos(\theta_1 - \theta_2)$ — ②

① - ② $\Rightarrow |1 - z_1\bar{z}_2| - |z_1 - z_2|^2 = 1 + (r_1r_2)^2 - r_1^2 - r_2^2$

$= (1 - r_1^2) + r_2^2(r_1^2 - 1)$

$= (1 - r_1^2)(1 - r_2^2)$

$= (1 - |z_2|^2)(1 - |z_1|^2)$

$1 + \sin\theta = x \qquad i\cos\theta = yi$

$\left(\frac{x + yi}{x - yi}\right) = \frac{x^2 - y^2}{x^2 + y^2} + \frac{2xyi}{x^2 + y^2}$

$= \frac{2\sin\theta(1 + \sin\theta)}{2(1 + \sin\theta)} + \frac{2i(1 + \sin\theta)\cos\theta}{2(1 + \sin\theta)}$

$= \sin\theta + i\cos\theta$

$\frac{\left(1 + \sin\frac{\pi}{5} + i\cos\frac{\pi}{5}\right)^5}{\left(1 + \sin\frac{\pi}{5} - i\cos\frac{\pi}{5}\right)^5} = \left(\sin\frac{\pi}{5} + i\cos\pi/5\right)^5$

$= \sin\pi + i\cos\pi$

$= (-i)$ — ①

$\left(1 + \sin\frac{\pi}{5} + i\cos\frac{\pi}{5}\right)^5 = -i\left[1 + \sin\frac{\pi}{5} - i\cos\frac{\pi}{5}\right]^5$

$\left(1 + \sin\frac{\pi}{5} + i\cos\frac{\pi}{5}\right)^5 + i\left(1 + \sin\frac{\pi}{5} - i\cos\frac{\pi}{5}\right)^5 = 0$

14) a) $f(x) = \frac{x}{(x-1)^2}$

$$f'(x) = \frac{(x-1)^2 \frac{dx}{dx} - x \frac{d}{dx}(x-1)^2}{(x-1)^4}$$

$$= \frac{(x-1)^2 - x \times 2(x-1)}{(x-1)^4}$$

$$= \frac{(x-1) - 2x}{(x-1)^3} = -\frac{(1+x)}{(x-1)^3} \quad (x \neq -1)$$

i) $y = \frac{x}{(x-1)^2}$

$x = 0 \Rightarrow y = 0$ $\therefore (0,0)$ ல் அச்சுக்களை வெட்டும்

ii) $y = \frac{x}{(x-1)^2}$

$x = 1$ ஆக $y \to \infty$ $\therefore x = 1$ நிலைக்குத்து அணுகுகோடு

$$\lim_{x \to \infty} y = \lim \frac{x}{x^2 - 2x + 1} = \lim_{x \to \infty} \frac{1/x}{1 - 2/x + 1/x^2} = 0$$

$y = 0$ கிடை அணுக கோடு

$f'(x) = 0$ எனில்

$-\frac{(1+x)}{(x-1)^3} = 0$ $x = -1$ திரும்புபுள்ளி

	$-\infty < x < -1$	$-1 < x < 1$	$1 < x < +\infty$
$f'(x)$	$(-)$	$(+)$	$(-)$
y	↘ குறையும்	↗ அதிகரிக்கும்	↘ குறையும்

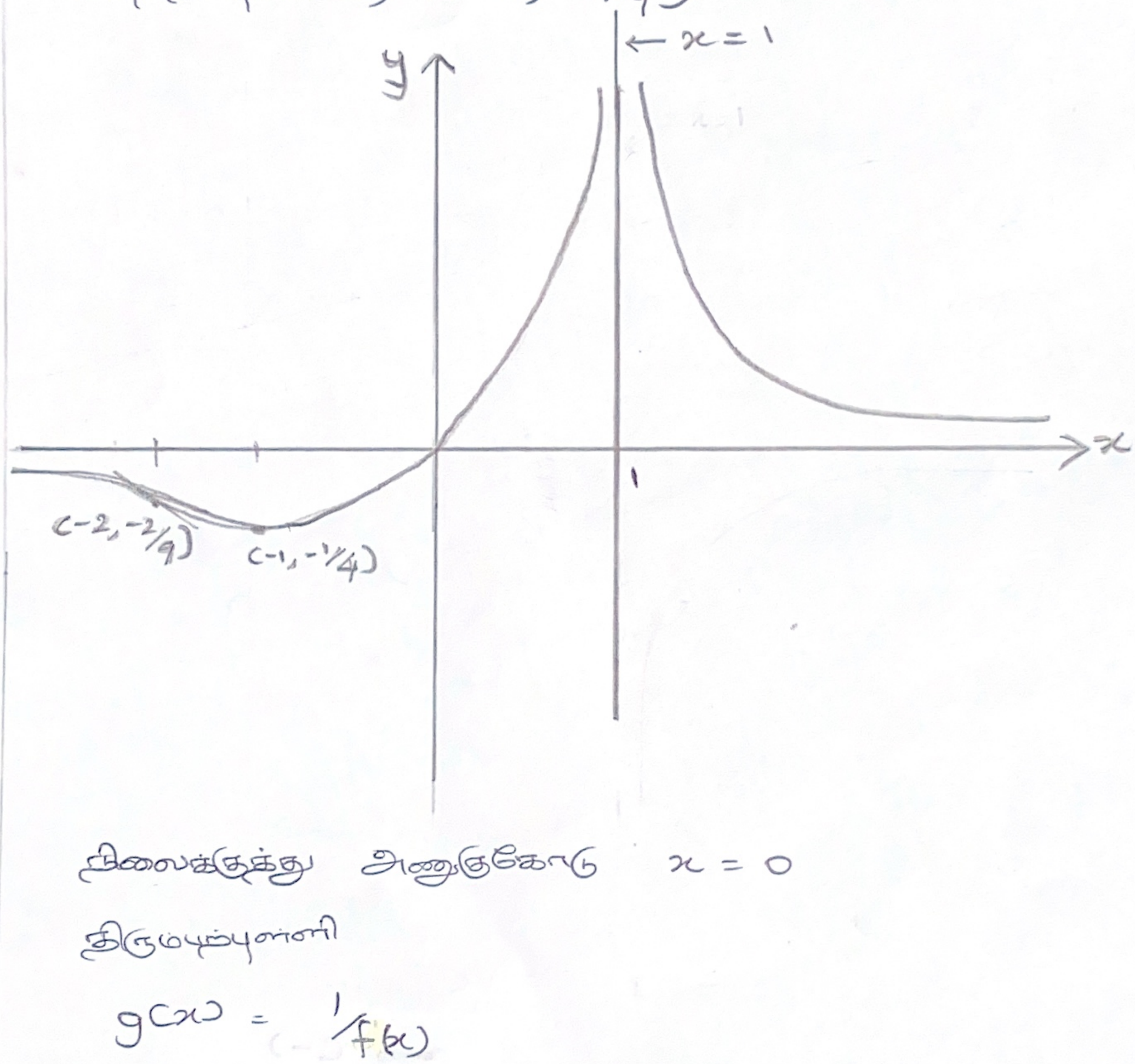
திரும்பு புள்ளி $\Rightarrow (-1, -1/4)$

$f''(x) = 0$ எனில்

$$\frac{2(x+2)}{(x-1)^4} = 0 \qquad x = -2 \text{ ஒரு விலக்குபுள்ளி}$$

	$-\infty < x < -2$	$-2 < x < 1$	$1 < x < \infty$
$f''(x)$	(-)	(+)	(+)
குழிவின் தன்மை	கீழ்நோக்கி	மேல்நோக்கி	மேல்நோக்கி

விலக்கு புள்ளி $\Rightarrow (-2, -2/9)$



நிலைக்குத்து அணுகுகோடு $x = 0$

திருப்புபுள்ளி

$$g(x) = 1/f(x)$$

$$\frac{dg(x)}{dx} = \frac{d\ 1/f(x)}{dx}$$

$$= \frac{d\ f(x)^{-1}}{dx}$$

$$= (-1)\, f(x)^{-2} f'(x)$$

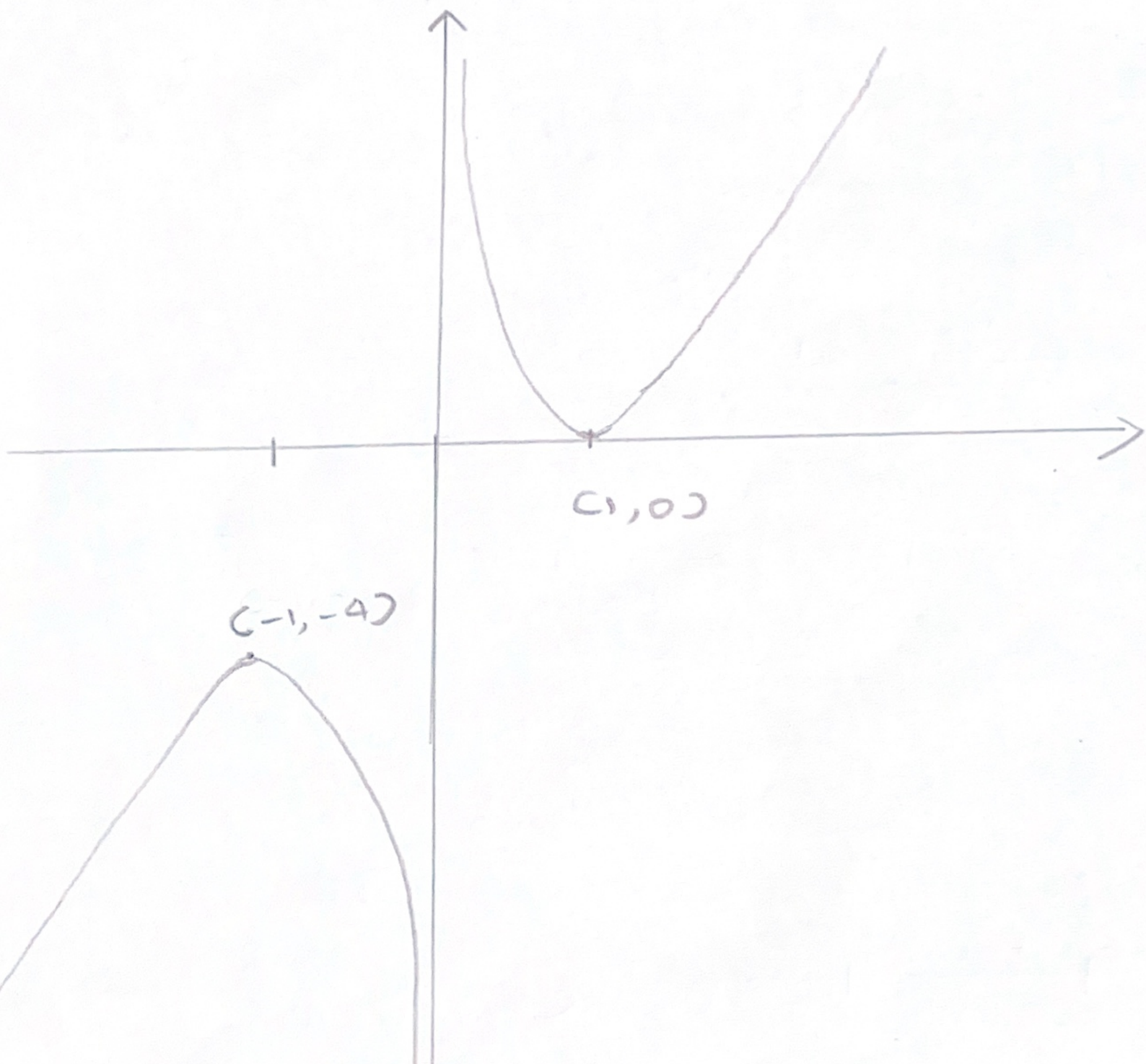
$$= \frac{-1}{f(x)^2} f'(x)$$

$$= \left[\frac{-(x-1)^4}{x^2}\right]\left[-\frac{(x+1)}{(x-1)^3}\right]$$

$g'(x) = \frac{x^2-1}{x^2}$ $\quad g'(x) = 0$

$x = 1$ $\quad x = -1$ இல்

திரும்பல் உண்டு



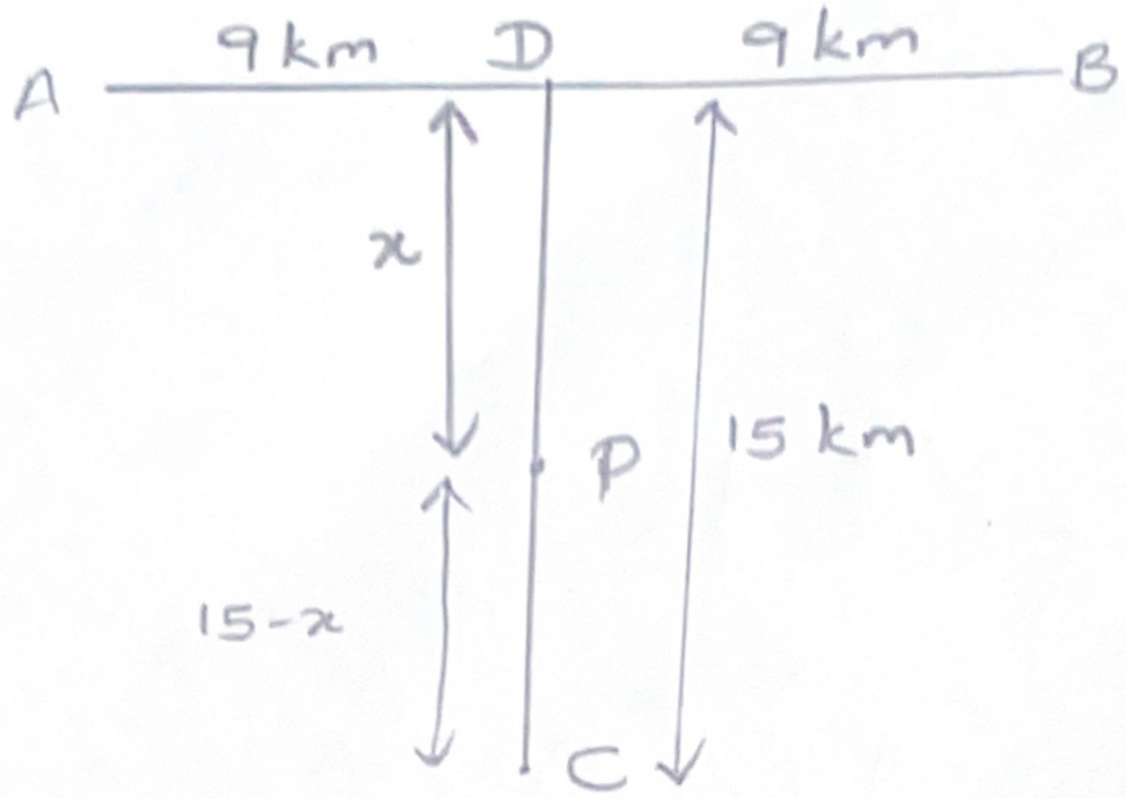
$f(x)^2 = 1$

$f(x) = \frac{1}{f(x)}$

இரு வளையுறைகளும் இரு இடங்களில் வெட்டுவதால்

$f(x)^2 = 1$ இற்கு இரு தீர்வுகள் உண்டு.

b)



$H(x) = CP + PA + PB$

$= (15-x) + \sqrt{9^2+x^2} + \sqrt{9^2+x^2}$

$= (15-x) + 2\sqrt{9^2+x^2}$

$H'(x) = \frac{d}{dx}(15-x) + 2\frac{d}{dx}\sqrt{81+x^2}$

$= -1 + 2 \times \frac{1}{2\sqrt{81+x^2}} \times 2x$

$= \frac{2x}{\sqrt{81+x^2}} - 1$

$H'(x) = 0$ எனில்

$\frac{2x}{\sqrt{81+x^2}} = 1$

$4x^2 = 81+x^2$

$3x^2 = 81$

$x^2 = 27$

$x = \pm 3\sqrt{3}$ $\quad x = 3\sqrt{3}$ km $(x>0)$

$x = 3\sqrt{3}$

$0 < x < 3\sqrt{3} \Rightarrow y < 0$

$x > 3\sqrt{3} \Rightarrow y > 0$

$\therefore [H(x)]_{\text{இழிவு}} = 15 - 3\sqrt{3} + 2\sqrt{81+27}$

$= 3[5+3\sqrt{3}]$ km

15) a) $x^2 = u$

$$\frac{du}{dx} = 2x$$

$$\int \frac{x}{x^4 - x^2 + 1}\, dx = \int \left(\frac{1}{2}\right)\left(\frac{1}{u^2 - u + 1}\right) du$$

$$= \frac{1}{2}\int \left[\frac{1}{(u - 1/2)^2 + 3/4}\right] du$$

$$= \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)\int \left[\frac{\sqrt{3}/2}{(u - 1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)}\right] du$$

$$= \frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{u - 1/2}{\sqrt{3}/2}\right)\right]$$

$$= \frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x^2 - 1}{\sqrt{3}}\right)\right] + C$$

b) $\sin x + \cos x = \sqrt{2}\left[\cos\left(\frac{\pi}{4} - x\right)\right]$

$R = \sqrt{2} \qquad \alpha = x$

$$\int_0^{\pi/2} \left[\frac{1}{2\sin x \cos x + 1}\right] dx = \int_0^{\pi/2} \frac{1}{[\sin x + \cos x]^2}\, dx$$

$$= \int_0^{\pi/2} \frac{1}{2}\sec^2\left(x - \frac{\pi}{4}\right) dx$$

$$= \frac{1}{2}\left[\tan\left(x - \frac{\pi}{4}\right)\right]_0^{\pi/2}$$

$$= \frac{1}{2}[1 + 1] = 1$$

$$\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$$

i) $\int_0^{\pi/2} \frac{\sin^2 x}{(\sin x + \cos x)^2}\, dx = \int_0^{\pi/2} \frac{\cos^2 x}{(\sin x + \cos x)^2}\, dx = I$

$$2I = \int_0^{\pi/2} \frac{1}{(\sin x + \cos x)^2}\, dx \qquad I = \frac{1}{2} \times 1 = \frac{1}{2}$$

ii) $I = \int_0^{\pi/2} \frac{x}{(\sin x + \cos x)^2} dx$

$= \int_0^{\pi/2} \frac{\pi/2 - x}{(\sin x + \cos x)^2} dx$

$= \int_0^{\pi/2} \frac{\pi/2}{(\sin x + \cos x)^2} dx - \int_0^{\pi/2} \frac{x}{(\sin x + \cos x)^2} dx$

$2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{1}{(\sin x + \cos x)^2} dx$

$I = \frac{\pi}{2} \times \frac{1}{2} \times 1 = \pi/4$

c) $\int_0^1 x \ln(1+x^2) dx = \int_0^1 [\ln(1+x^2)] \left[\frac{d(x^2/2)}{dx}\right] dx$

$= \left[\frac{x^2}{2} \ln(1+x^2)\right]_0^1 - \int_0^1 \frac{x^2}{2} \left(\frac{1}{1+x^2}\right)(2x) dx$

$= \frac{1}{2} \ln 2 - \int_0^1 \left(\frac{x^3}{x^2+1}\right) dx$

$= \frac{1}{2} \ln 2 - \int_0^1 \frac{(x^3+x) - x}{x^2+1} dx$

$= \frac{1}{2} \ln 2 - \int_0^1 x\, dx + \frac{1}{2} \int_0^1 \frac{2x}{x^2+1} dx$

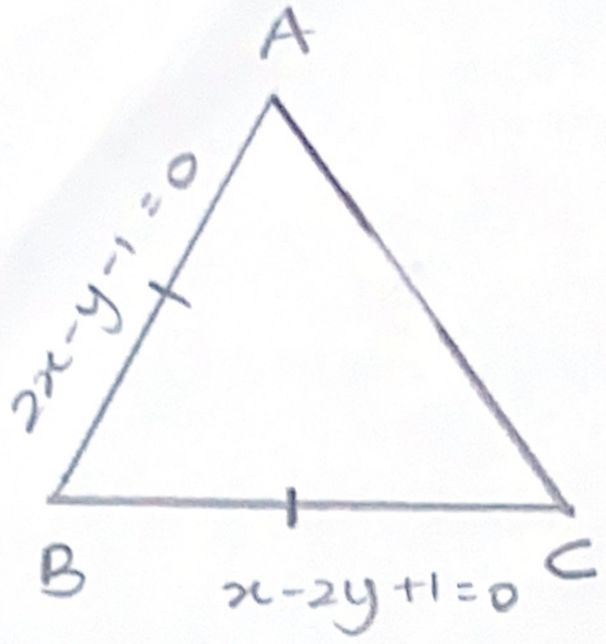
$= \frac{1}{2} \ln 2 - \left[\frac{x^2}{2}\right]_0^1 + \frac{1}{2} \left[\ln(x^2+1)\right]_0^1$

$= \frac{\ln 2}{2} - \frac{1}{2} + \frac{1}{2} \ln 2$

$= \ln 2 - \frac{1}{2}$

16)

a)



$$\frac{2x-y-1}{\sqrt{4+1}} = \frac{\pm\,|x-2y+1|}{\sqrt{1+4}}$$

(+) $\Rightarrow$ $2x-y-1 = x-2y+1$

$$x+y-2=0 \quad \text{—①}$$

(-) $\Rightarrow$ $2x-y-1 = -x+2y-1$

$$3x-3y=0$$

$$x-y=0 \quad \text{—②}$$

② வது சமன்பாடு $\left(\frac{5}{2},\frac{5}{2}\right)$

புள்ளியை திருப்திசெய்வதால் $x-y=0$ அகக்கோண இருசமவெட்டியின் சமன்பாடாகும்.

AC யின் படித்திறன் $= -1$

$$-1 = \frac{5/2-y}{5/2-x} \qquad \frac{5}{2}-x = \frac{-5}{2}+y \qquad y+x-5=0$$

b) $S \equiv x^2+y^2-16+\lambda(4x-16y-9)=0$

$$S \equiv x^2+y^2+4\lambda x-16\lambda y-9\lambda = 0$$

வையம் $= (-2\lambda, 8\lambda)$

$(-2\lambda, 8\lambda)$ ஆகியன $2x+3y+5=0$ இவை திருப்தியாக்கும்.

$$2(-2\lambda)+3(8\lambda)+5=0$$

$$-4\lambda+24\lambda+5=0$$

$$20\lambda+5=0$$

$$\lambda = (-1/4)$$

$$S \equiv x^2+y^2+4\left(\frac{-1}{4}\right)x-16\left(\frac{-1}{4}\right)y-9\left(\frac{1}{4}\right)-16=0$$

$$x^2+y^2-x+4y-\frac{73}{4}=0$$

$S_2 = x^2 + y^2 + 2gx + 2fy + c = 0$

x அச்சை தொடுவதால் $y = 0$ என பிரதியிடும்

போது பொருந்தும் மூலம் பெறப்படல்

$x^2 + 2gx + c = 0$ — ①

$b^2 - 4ac = 0$

$4g^2 - 4c = 0$

(3,0) இனை திருப்தியாக்கும்

① $\Rightarrow 9 + 6g + c = 0$ — ②

$g^2 = c$ — ③

③, ② $\Rightarrow$

$g^2 + 6g + 9 = 0$

$(g+3)^2 = 0$

$g = -3$ $\quad c = 9$

$S_2 = x^2 + y^2 - 6x + 2fy + 9 = 0$

நிமிர் கோணத்தில் வெட்டுன்

$2gg' + 2ff' = c + c'$

$2(-3)\left(\frac{-1}{2}\right) + 2f(2) = 9 - \frac{73}{4}$

$4f = 6 - \frac{73}{4}$

$f = \frac{-49}{16}$

$S_2 = x^2 + y^2 - 6x - \frac{49}{8}y + 9 = 0$

17)

a) $\sin(A+B) = \sin A\cos B + \cos A\sin B$ — ①

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ — ②

$$\frac{①}{②} \Rightarrow \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$= \frac{\tan \pi/4 + \tan \pi/6}{1 - \tan \pi/4 \tan \pi/6}$$

$$= \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \qquad = \frac{(\sqrt{3}+1)^2}{3-1} \qquad = \sqrt{3}+2$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 - \tan A\tan B}$$

$$\tan(\pi/12) = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

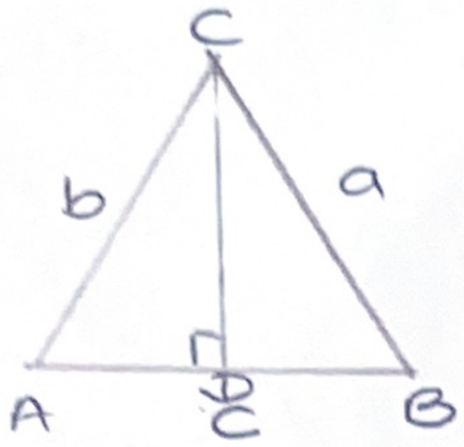
$$= \frac{\tan \pi/4 - \tan \pi/6}{1 + \tan \pi/4 \tan \pi/6}$$

$$= \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}}$$

$$= \frac{(\sqrt{3}-1)^2}{2}$$

$$= 2-\sqrt{3}$$

யாதேனும் முக்கோணம் ABC ல்



$CD = b\sin A \qquad BD = c - b\cos A$

Δ ABC ல்

$BC^2 = CD^2 + BD^2$

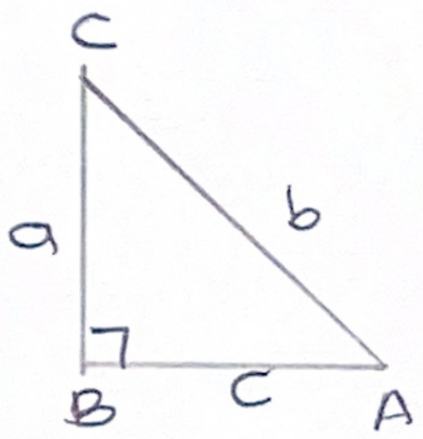
$a^2 = b^2\sin^2 A + (c - b\cos A)^2$

$a^2 = b^2\sin^2 A + b^2\cos^2 A + c^2 - 2bc\cos A$

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \frac{b^2+c^2-a^2}{2bc}$ இவ்வாறே $\cos B = \frac{a^2+c^2-b^2}{2ac}$

$\cos C = \frac{b^2+a^2-c^2}{2ab}$

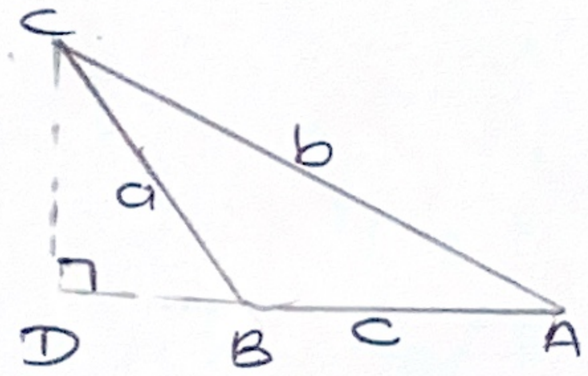


$b^2 = a^2 + c^2$

$b^2 = a^2 + c^2 - 2ac\cos\frac{\pi}{2}$

$b^2 = a^2 + c^2 - 2ac\cos B \qquad \cos B = \frac{a^2+c^2-b^2}{2ac}$

இவ்வாறே $\cos A = \frac{b^2+c^2-a^2}{2bc} \qquad \cos C = \frac{a^2+b^2-c^2}{2ab}$



$b^2 = (a\sin B)^2 + [c - a\cos B]^2$

$b^2 = a^2 + c^2 - 2ac\cos B$

இவ்வாறே

$\cos B = \frac{a^2+c^2-b^2}{2ac} \qquad \cos A = \frac{b^2+c^2-a^2}{2bc} \qquad \cos C = \frac{a^2+b^2-c^2}{2ab}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = b^2 + c^2 - 2bc\left[1 - 2\sin^2\left(\frac{A}{2}\right)\right]$

$= b^2 + c^2 - 2bc + 4bc\sin^2\left(\frac{A}{2}\right)$

$a^2 = (b-c)^2 + 4bc\sin^2\left(\frac{A}{2}\right)$ —— (R_1)

$a = (b-c)\sec\phi$ —— (R_2)

(R1), (R2) $\Rightarrow$ $(b-c)^2\sec^2\phi = (b-c)^2 + 4bc\sin^2\left(\frac{A}{2}\right)$

$(b-c)^2(\sec^2\phi - 1) = 4bc\sin^2\left(\frac{A}{2}\right)$

$(b-c)^2\tan^2\phi = 4bc\sin^2\left(\frac{A}{2}\right)$

$\tan\phi = \frac{2\sqrt{bc}\sin\left(\frac{A}{2}\right)}{(b-c)}$

---

c) $\tan^{-1}(2x+1) + \tan^{-1}(2x-1) = \tan^{-1}(2)$

$\tan^{-1}(2x+1) = A$

$\tan^{-1}(2x-1) = B$

$A + B = \tan^{-1}(2)$

$\tan(A+B) = 2$

$\frac{\tan A + \tan B}{1 - \tan A\tan B} = 2$

$\frac{2x+1+2x-1}{1-(2x+1)(2x-1)} = 2$

$2x = 1 - (4x^2 - 1)$

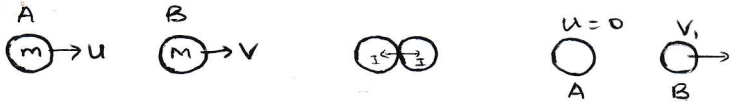
$4x^2 + 2x - 2 = 0$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$x = \frac{1}{2}$ or $x = (-1)$

1)



தொகுதிக்கு

$\rightarrow$
$I = \Delta mv$

$0 = MV_1 - (MV + mu)$

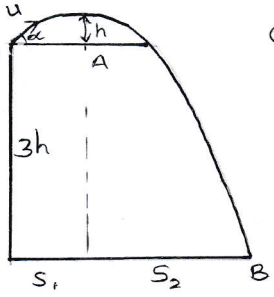
$V_1 = \frac{MV + mu}{M}$ —— ①

$e = \frac{V_1}{u - V}$

① $\Rightarrow$ $e = \frac{MV + mu}{M(u - v)}$

---

2)



(O → A) $\uparrow V^2 = u^2 + 2as$

$0 = u^2\sin^2\alpha - 2gh$

$u^2 = \frac{2gh}{\sin^2\alpha}$ —— ①

(O → A) $\uparrow V = u + at$

$0 = u\sin\alpha - gt_1$

$t_1 = \frac{u\sin\alpha}{g}$ —— ②

(O → A) $\rightarrow S = ut + \frac{1}{2}at^2$

$S_1 = u\cos\alpha\left(\frac{u\sin\alpha}{g}\right) = \frac{u^2\cos\alpha\sin\alpha}{g}$

$= \frac{2gh\cos\alpha\sin\alpha}{\sin^2\alpha\, g}$

$= 2h\cot\alpha$

(A → B)

$\downarrow S = ut + \frac{1}{2}at^2$

$4h = \frac{1}{2}gt^2$

$t^2 = \frac{8h}{g}$

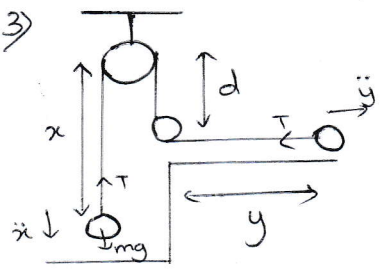
(A → B) $\rightarrow S = ut + \frac{1}{2}at^2$

$S_2 = u\cos\alpha\left(\sqrt{\frac{8h}{g}}\right) = \left(\sqrt{\frac{2gh}{\sin^2\alpha}}\right)\left(\sqrt{\frac{8h}{g}}\right)\cos\alpha$

$= 4h\cot\alpha$

$R = S_1 + S_2 = 6h\cot\alpha$

3)



$x + d + y = \ell$

$\ddot{x} + \ddot{y} = 0$ — ①

$\downarrow 2mg - T = 2m\ddot{x}$

$\ddot{x} = \left(g - \frac{T}{2m}\right)$ — ②

$\leftarrow F = ma$

$T = m(-\ddot{y})$ $\qquad \ddot{y} = \left(-\frac{T}{m}\right)$ — ③

$-\frac{T}{m} + g - \frac{T}{2m} = 0$

$\frac{3T}{2m} = g$

$T = \frac{2mg}{3}$ — ④

$\downarrow s = ut + \frac{1}{2}at^2$

$h = 0 + \frac{1}{2}\left(g - \frac{g}{3}\right)t^2$

$= \frac{1}{2}\left(\frac{2g}{3}\right)t^2$

$t = \sqrt{\frac{3h}{g}}$

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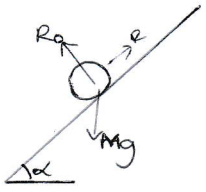
4)



$(M) \swarrow F = ma$

$Mg\sin\alpha - R = MF$ — ①

$\swarrow s = ut + \frac{1}{2}at^2$

$d = 0 + \frac{1}{2}(F)(1)$

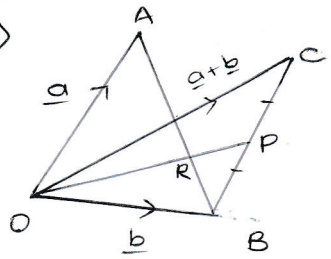
$2d = F$ — ②

①, ② ⇒

$Mg\sin\alpha - R = (2d)M$

$R = M(g\sin\alpha - 2d)$

5)



$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \underline{a} + \underline{b} - \underline{b} = \underline{a}$

$\overrightarrow{BP} = \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}\underline{a}$

ΔOPB

$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP}$

$= \underline{b} + \frac{1}{2}\underline{a}$

$= \frac{1}{2}(\underline{a} + 2\underline{b})$

$\overrightarrow{OR} = \mu(\overrightarrow{OP})$ $\quad \mu \in R$

ΔORB ου

$\overrightarrow{OR} + \overrightarrow{RB} = \overrightarrow{OB}$

$\frac{\mu}{2}(\underline{a} + 2\underline{b}) + \overrightarrow{RB} = \underline{b}$

$\overrightarrow{RB} = \underline{b} - \frac{\mu}{2}(\underline{a} + 2\underline{b})$

$\frac{\mu}{2} = k$ στους $\quad R = \underline{b} - k(\underline{a} + 2\underline{b})$

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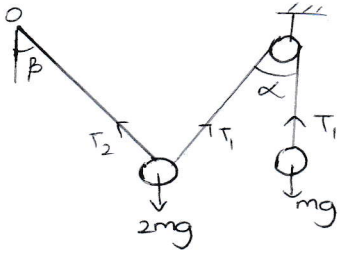
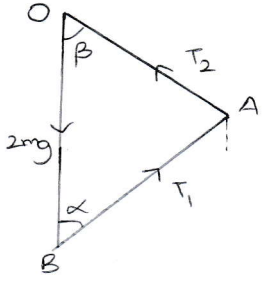
6)





$T_1 = mg$

ΔOAB ου sin σε λόγους

$\frac{mg}{\sin\beta} = \frac{T_2}{\sin\alpha} = \frac{2mg}{\sin[180-(\alpha+\beta)]}$

$\frac{mg}{\sin\beta} = \frac{2mg}{\sin(\alpha+\beta)}$

$2\sin\beta = \sin(\alpha+\beta)$

$2\sin\beta = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$2 = \sin\alpha\cot\beta + \cos\alpha$

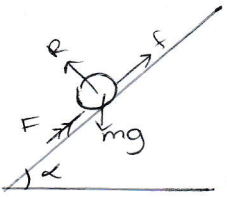
$2 - \cos\alpha = \sin\alpha\cot\beta$

$\cot\beta = \frac{2-\cos\alpha}{\sin\alpha}$

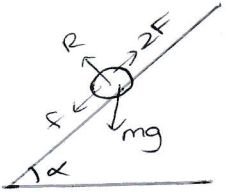
$\tan\beta = \frac{\sin\alpha}{2-\cos\alpha}$

$\beta = \tan^{-1}\left(\frac{\sin\alpha}{2-\cos\alpha}\right)$

7)



கீழ்நோக்கி வழுக்கும் போது
(I)



மேல்நோக்கி அனைக்கும் போது
(II)

(I) $F + f = mg\sin\alpha$

$F + \mu mg\cos\alpha = mg\sin\alpha$

$F = mg(\cos\alpha - \sin\alpha)$

(II) $f + mg\sin\alpha = 2F$

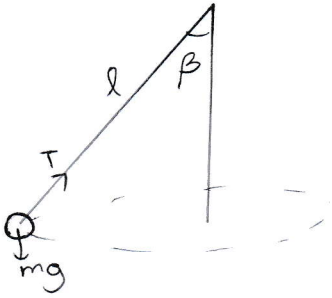
$\mu mg\cos\alpha + mg\sin\alpha = 2mg\sin\alpha - 2\mu mg\cos\alpha$

$3\mu\cos\alpha = \sin\alpha$

$\mu = \frac{1}{3}\tan\alpha$

---

8)



$\uparrow F = ma$

$T\cos\beta = mg$

$T = \frac{mg}{\cos\beta}$

$\rightarrow F = ma$

$T\sin\beta = mr\omega^2$

$mg\frac{\sin\beta}{\cos\beta} = ml\sin\beta\omega^2$

$g = \omega^2 l\cos\beta$

9) $P(A) = \frac{3}{5}$ $P(B) = \frac{2}{3}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{5} + \frac{2}{3} - P(A \cap B)$

ஆனால்

$\frac{2}{3} \leq P(A \cup B) \leq 1$

$\frac{2}{3} \leq \frac{3}{5} + \frac{2}{3} - P(A \cap B) \leq 1$

$\frac{2}{3} \leq \frac{3}{5} + \frac{2}{3} - P(A \cap B)$ $\frac{19}{15} - P(A \cap B) \leq 1$

$P(A \cap B) \geq \frac{4}{15}$

$P(A \cap B) \leq \frac{3}{5}$

$\therefore \frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$

---

10) வகுப்பாயிடையின் பருமன் $2C$ எனக்க

$(40 - 6C) - (40 - 4C)$ 22.5

$(40 - 4C) - (40 - 2C)$

$(40 - 2C) - 40$

7
19) 8
27
15) 12
2

$\frac{40 - 6C + 40 - 4C}{2} = 22.5$

$C = 3.5$

வகுப்பாயிடையின் பருமன் = 7

ஆகார வகுப்பு $= (40 - 6C) - (40 - 4C)$

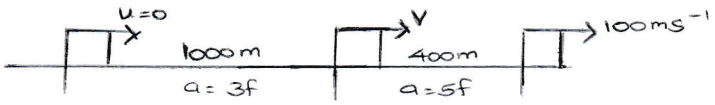
$= 19 - 26$

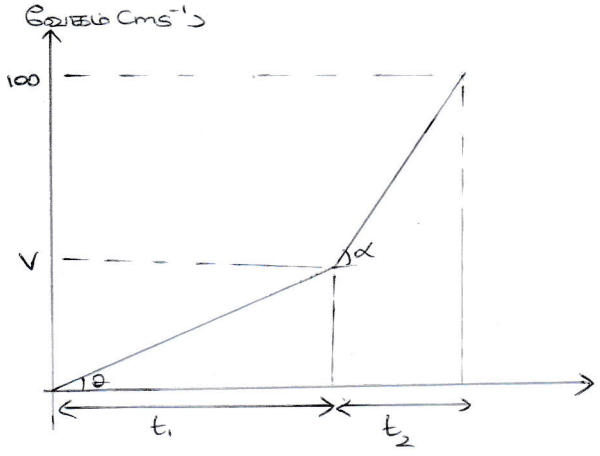
ஆகாரம் $= L + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) C$

$= 19 + \frac{8}{20} \times 7$

$= 21.8$

11) a)





$\tan\theta = 3f$ $\quad \tan\alpha = 5f$

$3f = \frac{V}{t_1}$ — ① $\quad 5f = \frac{100-V}{t_2}$ — ②

$\frac{①}{②} \Rightarrow \frac{3}{5} = \left(\frac{V}{t_1}\right)\left(\frac{t_2}{100-V}\right)$

$\frac{3}{5} = \left(\frac{V^2}{2000}\right)\left(\frac{800}{100^2-V^2}\right)$

$3\times100^2 - 3V^2 = 2V^2$

$5V^2 = 3\times100\times100$

$V^2 = 6\times10^3$

$V = 10\sqrt{60}$

$= 20\sqrt{15}\ ms^{-1}$

$1000 = \frac{1}{2}t_1 V \qquad t_1 = \frac{2000}{V}$

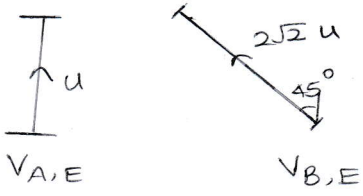
$400 = \frac{1}{2}(V+100)t_2 \qquad t_2 = \frac{800}{V+100}$

$t_1 = \frac{2000}{20\sqrt{15}} = \frac{100}{\sqrt{15}}$

$t_2 = \frac{800}{20\sqrt{15}+100} = \frac{40}{\sqrt{15}+5}$

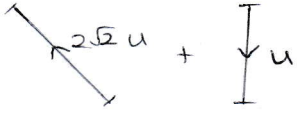
மொத்த நேரம் $= t_1 + t_2 = \frac{100}{\sqrt{15}} + \frac{40}{5+\sqrt{15}} = 20\left[\frac{5}{\sqrt{15}} + \frac{2}{(5+\sqrt{15})}\right]$

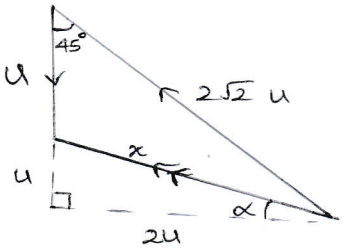
b)



$V_{A,E}$ $\qquad$ $V_{B,E}$

$V_{B,A} = V_{B,E} + V_{E,A}$

$=$ 

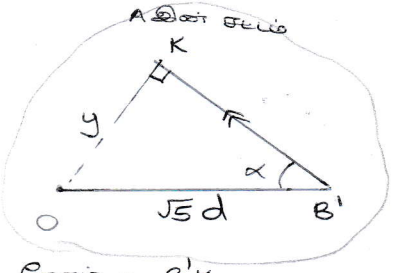


$$x = \sqrt{4u^2 + u^2}$$

$$= \sqrt{5}\,u$$

$$\sin\alpha = \frac{u}{\sqrt{5}\,u} = \frac{1}{\sqrt{5}}$$

i)



$$\sin\alpha = \frac{y}{\sqrt{5}\,d} = \frac{1}{\sqrt{5}}$$

$$y = d$$

மிகக்குறைந்த தூரம் = d km

நேரம் = $\frac{B'K}{V_{B,A}}$

$B'K = \sqrt{5}\,d\cos\alpha$

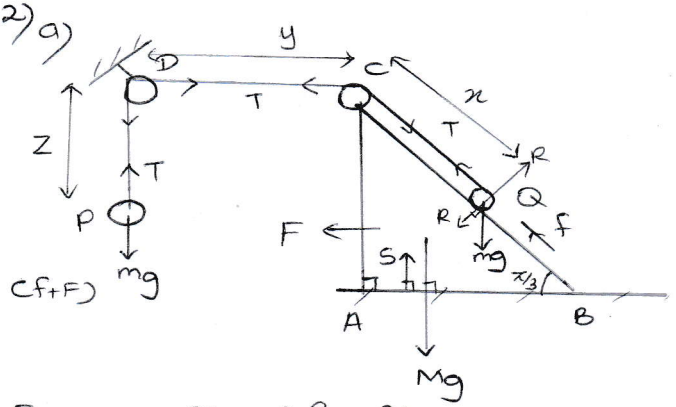
$= \sqrt{5}d \times \frac{2}{\sqrt{5}} = 2d$ km

$= \frac{2d \text{ km}}{\sqrt{5}\,u \text{ km/h}}$

$= \frac{2d}{\sqrt{5}\,u}$ h

ii) நேரம் = $\frac{2d}{\sqrt{5}\,u} \times 60 = \frac{24\sqrt{5}\,d}{u}$ நிமிடம்

12) a)



$x + y + z$ = மாறிலி

$\ddot{z} = -\ddot{x} - \ddot{y}$

$= f + F$

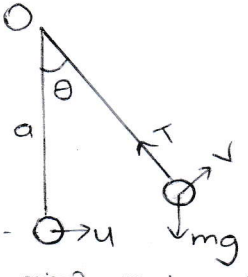
$F = ma$ ஐ பிரயோகிக்க

(P) $\downarrow$ $mg - T = m(F+f)$ — ①

(Q) $\nearrow$ $T - mg\sin\frac{\pi}{3} = m\left(f + F\cos\frac{\pi}{3}\right)$ — ②

(Q+M) $\leftarrow$ $T = MF + m\left(F + \cos\frac{\pi}{3}\right)$ — ③

b)



$I = mu$

Z.P.L - - -

பொறிமுறை சக்தி காப்பு கோட்பாடு

$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mg(a - a\cos\theta)$

$v^2 = u^2 - 2ga(1 - \cos\theta)$

$\nearrow$ $F = ma$

$T - mg\cos\theta = \frac{mv^2}{a}$

$T = \frac{m}{a}\left[u^2 - 2ga + 2ga\cos\theta + ag\cos\theta\right]$

$= \frac{mu^2}{a} - 2mg + 3mg\cos\theta$

$= \frac{I^2}{ma} - 2mg + 3mg\cos\theta$

i) துணிக்கை முழுவட்டத்தை ஆக்குவதால் $\theta = \pi$ ஆக $T > 0$

$$T = \frac{I^2}{ma} - 2mg + 3mg\cos\pi$$

$$= \frac{I^2}{ma} - 5mg$$

$$\frac{I^2}{ma} - 5mg > 0 \qquad I^2 > 5m^2ga$$

$$I > m\sqrt{5ag}$$

ii) $\frac{\pi}{2} < \theta < \pi$

$\theta = \frac{\pi}{2}$ ஆக $T_1 = \frac{I^2}{ma} - 2mg + 3mg\cos\frac{\pi}{2} = \frac{I^2}{ma} - 2mg$

$\theta = \pi$ ஆக $T_2 = \frac{I^2}{ma} - 2mg + 3mg\cos\pi = \frac{I^2}{ma} - 5mg$

α ஆக்கும் போது விலகுவதால் $T_1 > 0$ $T_2 < 0$

$I^2 > 2m^2ag$ $I^2 < 5m^2ag$

$I > m\sqrt{2ag}$ $I < m\sqrt{5ag}$

$\therefore m\sqrt{2ag} < I < m\sqrt{5ag}$

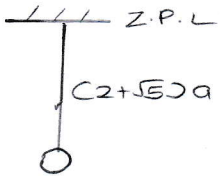
$\theta = 180 - \alpha$ ஆக $T = 0$

$$0 = \frac{I^2}{ma} - 2mg + 3mg\cos(180-\alpha)$$

$$3mg\cos\alpha = \frac{I^2}{ma} - 2mg$$

$$\cos\alpha = \frac{I^2}{3m^2ag} - \frac{2}{3}$$

13)



சக்தி காப்பு தத்துவப்படி

$$\frac{1}{2}mg\frac{(1+\sqrt{5})a^2}{a} - mg(2+\sqrt{5})a = \frac{1}{2}mg\frac{(x-a)^2}{a} + mgx$$

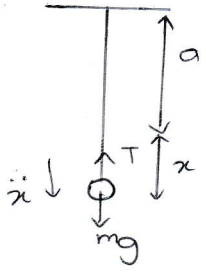
$$(1+\sqrt{5})a^2 - 2(2+\sqrt{5})a^2 = (x-a)^2 + 2ax$$

$$(6+2\sqrt{5})a^2 - (4+2\sqrt{5})a^2 = x^2 - 2ax + a^2 + 2xa$$

$$0 = x^2 - a^2$$

$$(x-a)(x+a) = 0$$

$$x = a$$



$\uparrow F = ma$

$T - mg = ma$

$$\left(\frac{mg}{a}\right)x - mg = m(-\ddot{x})$$

$$g - \frac{gx}{a} = \ddot{x}$$

$$\frac{g}{a}(a-x) = \ddot{x}$$

$a - x = X$ எனில் $\ddot{X} = -\ddot{x}$

$$\frac{g}{a}X = -\ddot{X}$$

$$\ddot{X} = -\frac{g}{a}X$$

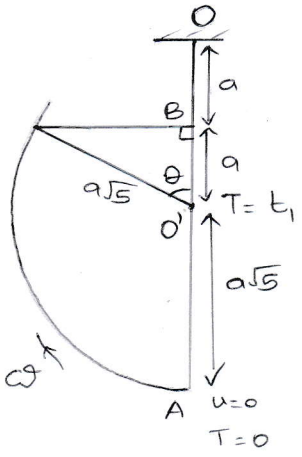
$$= -\omega^2 X \quad \left(\text{இங்கே } \omega = \sqrt{\frac{g}{a}}\right)$$

i) $X = 0$

$x = 0$

ii) S.H.M

iii) $T = 2\pi\sqrt{\frac{a}{g}}$



விசை $= a\sqrt{5}$

புள்ளி A யில் வேகம் 0 என்பதால்

$\theta = \omega t_1$

$(\pi - \theta) = \omega t_1$

$\pi - \cos\left(\frac{1}{\sqrt{5}}\right) = \omega t_1$

$t_1 = \frac{1}{\omega}(\pi - \theta)$ —— ①

B க்கு

$V^2 = \omega^2(A^2 - x^2)$

$V^2 = \frac{g}{a}(5a^2 - a^2)$

$V^2 = 4ag$

$V = 2\sqrt{ag}$ —— ②

B ல்

$x = 0 \quad X = a \quad A = a\sqrt{5}$

$\omega = \sqrt{g/a}$

$t_2 = 2\sqrt{\frac{a}{g}}$ —— ③

கீழிருந்து மேலே சென்ற

$t_1 + t_2 = \frac{1}{\omega}(\pi - \theta) + 2\sqrt{\frac{a}{g}} = \sqrt{\frac{a}{g}}(\pi - \theta + 2)$

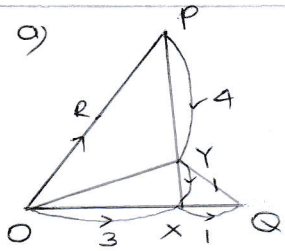
திரும்ப கீழே வர எடுக்கும் நேரம் $= t_1 + t_2$

மொத்த நேரம் $= 2(t_1 + t_2)$

$= \sqrt{\frac{a}{g}}(2\pi - 2\theta + 4)$

$= \sqrt{\frac{a}{g}}(2\pi - 2\beta + 4)$

14) a)



i) $\overrightarrow{OR} = \frac{1}{2}\overrightarrow{OP} = \frac{1}{2}\underline{p}$

$\overrightarrow{OX} = \frac{3}{4}\overrightarrow{OQ} = \frac{3}{4}\underline{q}$

$\triangle OPX$ ல்

$\overrightarrow{PX} = \overrightarrow{PO} + \overrightarrow{OX} = \frac{3}{4}\underline{q} - \underline{p}$

$\overrightarrow{YX} = \frac{1}{5}\overrightarrow{PX}$

$= \frac{1}{5}\left(\frac{3}{4}\underline{q} - \underline{p}\right)$

$\triangle OYX$ ல்

$\overrightarrow{OY} = \overrightarrow{OX} + \overrightarrow{XY}$

$= \frac{3}{4}\underline{q} - \frac{1}{5}\left(\frac{3}{4}\underline{q} - \underline{p}\right)$

$= \frac{3}{5}\underline{q} + \frac{1}{5}\underline{p} = \frac{1}{5}(3\underline{q} + \underline{p})$

ii) $\overrightarrow{XQ} = \frac{1}{4}\underline{q}$

$\triangle XYQ$ ல்

$\overrightarrow{QY} = \overrightarrow{QX} + \overrightarrow{XY} = -\frac{1}{4}\underline{q} - \frac{1}{5}\left(\frac{3}{4}\underline{q} - \underline{p}\right)$

$= \frac{1}{5}(\underline{p} - 2\underline{q})$

$\triangle QOR$ ல்

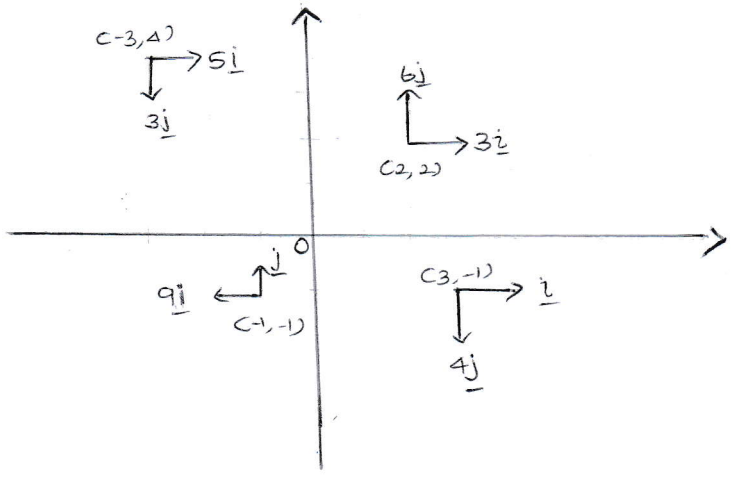
iii) $\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OR}$

$= -\underline{q} + \frac{1}{2}\underline{p} = \frac{1}{2}(\underline{p} - 2\underline{q}) = \frac{5}{2}\overrightarrow{QY}$

$\therefore$ Q, Y, R ஒரே நேர்கோட்டில் உள்ளன

$QY : YR = 2 : 3$

b) i)



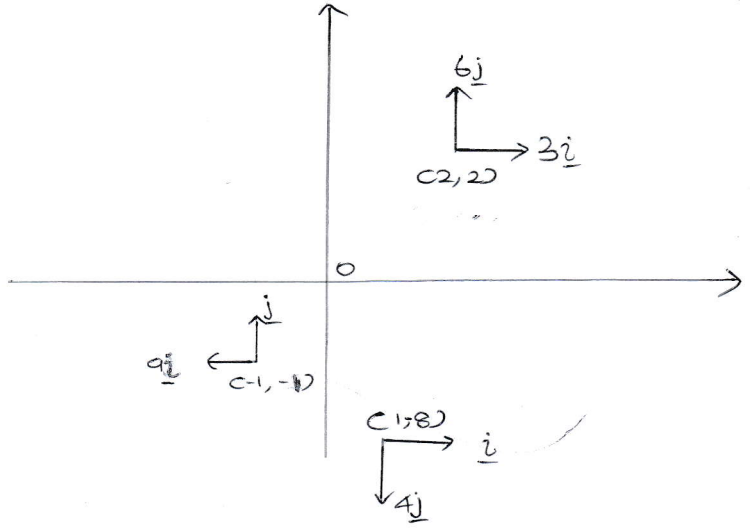
ii) $\uparrow Y = 6 - 4 + 1 - 3 = 0$

$\rightarrow X = 3 + 1 - 9 + 5 = 0$

O) $\Rightarrow 2\times6 - 4\times3 + 3\times3 - 2\times3 + 1(1) - 5(4) - (1)(1) - (9)(1)$

$= 12 - 12 + 9 - 6 + 1 - 20 - 1 - 9$

$= -26$ Nm



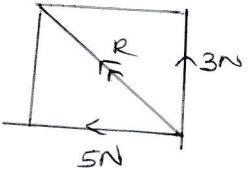
O) $6\times2 - 3\times2 + 8\times1 - 4\times1 - 1\times1 - 9\times1$

$= 12 - 6 + 8 - 4 - 1 - 9 = 0$ Nm

$\uparrow Y = 6 + 1 - 4 = 3$ N

$\rightarrow X = 3 + 1 - 9$

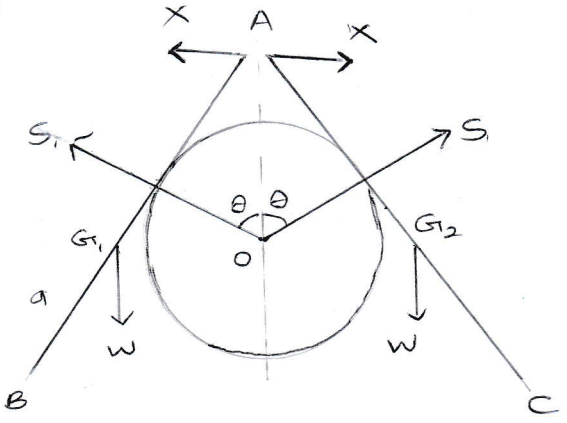
$= -5$ N



$R = \sqrt{3^2 + 5^2}$

$= \sqrt{34}$ N

15) a)



$(\overline{CAB})$ $A\curvearrowright$ $W(a\cos\theta) - S_1(r\tan\theta) = 0$

$$S_1 = \left(\frac{Wa\cos\theta}{r\tan\theta}\right) \text{—} (1)$$

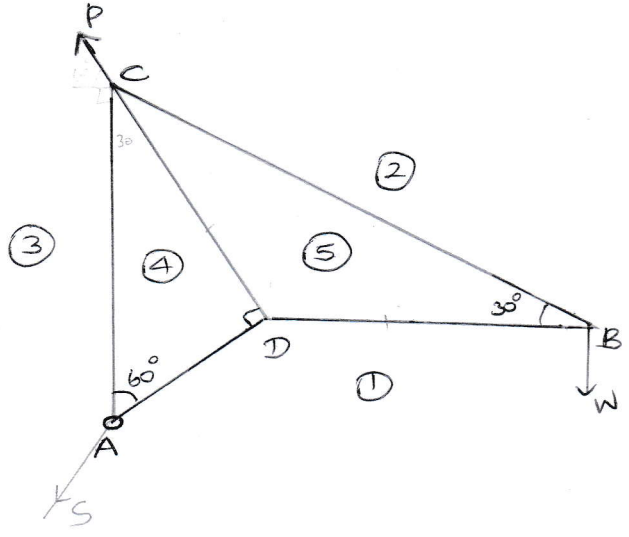
$\uparrow$ $S_1\cos\theta = W$

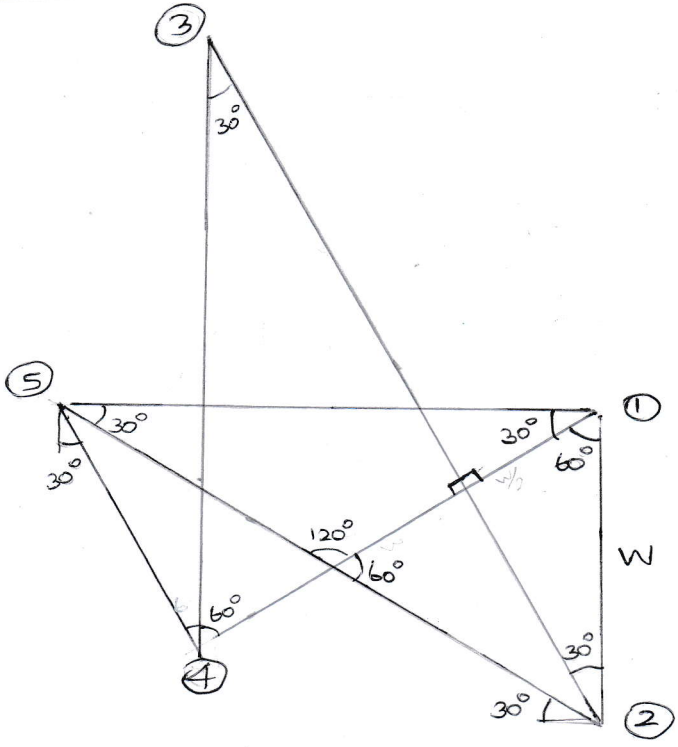
$$S_1 = \frac{W}{\cos\theta} \text{—} (2)$$

$(1), (2) \Rightarrow \frac{a\cos^2\theta}{r\sin\theta} = \frac{1}{\cos\theta}$

$$a\cos^3\theta = r\sin\theta$$

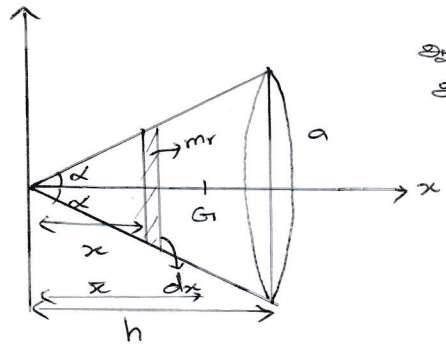
b)





கோல்	திருவை	உதைப்பு
AD	–	$3W/2$
AC	$2W$	–
DB	–	$\sqrt{3}W$
BC	$2W$	–
DC	–	$\frac{\sqrt{3}W}{2}$

16)



சமச்சீர் அச்சு x அச்சு ஆகுவினால் ஈர்ப்பு மையம் x அச்சின் அமையும்

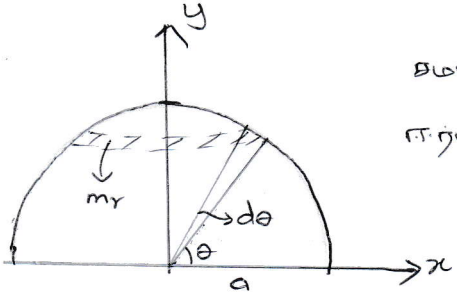
$m_r = \pi C x \tan\alpha)^2 dx\, \rho$

$x_r = x$

ρ - கனவளவு அடர்த்தி

$$\bar{x} = \frac{\sum_{x=0}^{h} m_r x_r}{\sum_{x=0}^{h} m_r} = \frac{\int_0^h \pi C x\tan\alpha)^2 dx\, \rho\, x}{\int_0^h \pi C x\tan\alpha)^2 dx\, \rho}$$

$$= \frac{\pi \tan^2\alpha\, \rho \int_0^h x^3 dx}{\pi \tan^2\alpha\, \rho \int_0^h x^2 dx} = \frac{\left[\frac{x^4}{4}\right]_0^h}{\left[\frac{x^3}{3}\right]_0^h} = \frac{3h}{4}$$



சமச்சீர் அச்சு y அச்சு எனபதால் ஈர்ப்பு மையம் y அச்சில் அமையும்

பரப்படர்த்தி - ρ

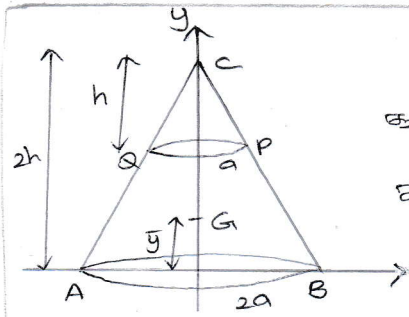
$m_r = 2\pi C a\cos\theta).a d\theta.\rho$

$y_r = a\sin\theta$

$$\bar{y} = \frac{\sum_{\theta=0}^{\pi/2} m_r y_r}{\sum_{\theta=0}^{\pi/2} m_r} = \frac{\int_0^{\pi/2} 2\pi a^2\cos\theta d\theta\, \rho\, a\sin\theta)}{\int_0^{\pi/2} 2\pi a^2\cos\theta d\theta\, \rho}$$

$$= \frac{a/2 \int_0^{\pi/2} \sin 2\theta\, d\theta}{\int_0^{\pi/2} \cos\theta\, d\theta}$$

$$= \frac{a\left[-\cos 2\theta/2\right]_0^{\pi/2}}{2\left[\sin\theta\right]_0^{\pi/2}} = \frac{a}{2}$$

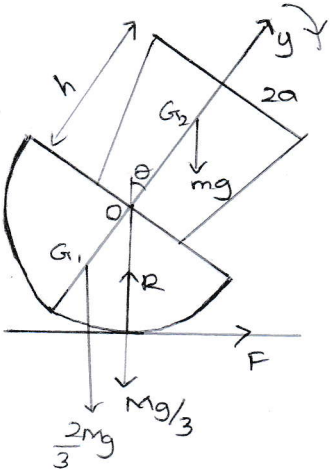


	கணமானவு அடர்த்தி ρ எண்க		
	உருவம்	திணிவு	yy அச்சிலிருந்து
கூம்பு ABC	$\frac{1}{3}\times C2a)^2 2h$		$\frac{h}{2}$
கூம்பு CPQ	$\frac{1}{3}\times a^2 h$		$\frac{h}{4}+h=\frac{5h}{4}$
எஞ்சிய அடர்	$\frac{7}{3}\times a^2 h$		$\bar{y}$

∴ அச்சு புள்ளி திருப்பம் எடுக்க

$$\frac{7}{3}\times a^2 h\bar{y} = \frac{8}{3}\times a^2 h\left(\frac{h}{2}\right) - \frac{1}{3}\times a^2 h\left(\frac{5h}{4}\right)$$

$$\bar{y} = \frac{11h}{28}$$



$$2\times C2a)^2 = 8\times a^2$$

$$\times C2a)^2 = 4\times a^2 \qquad 2:1$$

$$OG_2 = h - \frac{11h}{28}$$

$$= \frac{17h}{28}$$

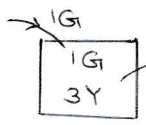
$$OG_1 = a$$

O↑ $\frac{2}{3}Mg a\sin\theta - mg\frac{17h}{28}\sin\theta \geq 0$

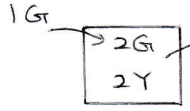
$$\frac{2}{3}Mg a \geq \frac{17h}{28}mg$$

$$\frac{M}{m} \geq \frac{51h}{56a}$$

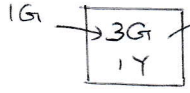
17) X – மஞ்சள் பந்து எடுத்தல்



$P(X_1) = \left(\frac{1}{4}\right)\left(\frac{3}{5}\right)$



$P(X_2) = \left(\frac{2}{4}\right)\left(\frac{2}{5}\right)$



$P(X_3) = \left(\frac{3}{4}\right)\left(\frac{1}{5}\right)$

$$P(X) = P(X_1 \cup X_2 \cup X_3)$$

$$= \left(\frac{1}{4}\right)\left(\frac{3}{5}\right) + \left(\frac{2}{4}\right)\left(\frac{2}{5}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{5}\right)$$

$$= \frac{3+4+3}{20}$$

$$P(X) = \frac{1}{2}$$

ii) $P\left(\frac{G_3}{X}\right) = \frac{P(G_3 \cap X)}{P(X)}$

$$= \frac{\left(\frac{3}{4}\right)\left(\frac{1}{5}\right)}{\left(\frac{10}{20}\right)}$$

$$= \frac{3}{10}$$

வகுப்பாயிடை	நடுப் பெறுமானம்	d	f	fd	fd^2
0 – 10	5	-3	5	-15	45
10 – 20	15	-2	15	-30	60
20 – 30	25	-1	45	-45	45
30 – 40	35	0	20	0	0
40 – 50	45	1	10	10	10
50 – 60	55	2	05	10	20
			100	-70	180

i) $\bar{x} = A + i\left(\frac{\Sigma fd}{n}\right)$

$= 35 + 10\left(\frac{-70}{100}\right) = 28$ புள்ளிகள்

2) $Me = L_0 + i\left[\frac{N/2 - f_c}{f}\right]$

$= 20 + 10\left[\frac{100/2 - 20}{45}\right]$

$= 26.67$ புள்ளி

3) $M_o = L_0 + i\left[\frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)}\right]$

$= 20 + 10\left[\frac{30}{30 + 25}\right]$

$= 20 + \frac{60}{11}$

$= 25.45$ புள்ளி

4) $6 = i\sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2}$

$= 10\sqrt{\frac{180}{100} - \left(\frac{-70}{100}\right)^2}$

$= \frac{10}{100}\sqrt{180 \times 100 - 70 \times 70} = \sqrt{180 - 49} = \sqrt{131}$

$= 11.445$ புள்ளிகள்

$= 11.45$ புள்ளி

II) $K = \frac{3(\bar{x} - M_o)}{6}$

$= \frac{3(28 - 25.45)}{11.45}$

$= \frac{7.65}{11.45} = 0.668$